Sept 6th

1. $\vec{u} = (1+t)\cdot\begin{pmatrix}1\\1-t\end{pmatrix} - t\cdot\begin{pmatrix}1\\-t\end{pmatrix} = \begin{pmatrix}1+t\\1-t^2\end{pmatrix} - \begin{pmatrix}t\\-t^2\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$

when will $\vec{u} \perp \begin{pmatrix}1\\2\end{pmatrix}$?

$\begin{pmatrix}1\\1\end{pmatrix}\cdot\begin{pmatrix}1\\2\end{pmatrix} = 1\times1 + 1\times2 = 3 \neq 0$ so $\vec{u}$ never $\perp \begin{pmatrix}1\\2\end{pmatrix}$

2. $\vec{v} = t\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + t^2\cdot\begin{pmatrix}0\\-1\\2\end{pmatrix} - \begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}t\\0\\0\end{pmatrix} + \begin{pmatrix}0\\-t^2\\2t^2\end{pmatrix} - \begin{pmatrix}0\\0\\1\end{pmatrix}$

$= \begin{pmatrix}t\\-t^2\\2t^2-1\end{pmatrix}$

when will $\vec{v} \parallel \begin{pmatrix}1\\2\\3\end{pmatrix}$?

$\vec{v} \times \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{vmatrix}\vec{e}_1 & \vec{e}_2 & \vec{e}_3\\ t & -t^2 & 2t^2-1\\ 1 & 2 & 3\end{vmatrix} = \begin{pmatrix}-t^2\times3 - 2\times(2t^2-1)\\ 2t^2-1 - 3t\\ 2t - (-t^2)\end{pmatrix} = \begin{pmatrix}-7t^2+2\\2t^2-3t-1\\t^2+2t\end{pmatrix}$

$\vec{v} \times \begin{pmatrix}1\\2\\3\end{pmatrix} = \vec{0} \Rightarrow \begin{cases}-7t^2+2=0 & ①\\ 2t^2-3t-1=0 & ②\\ t^2+2t=0 & ③\end{cases}$

there is no solution to t:

From ③ $t=0$ or -2

$t=0$ does not give ① or -2

so $\vec{v}$ never $\parallel \begin{pmatrix}1\\2\\3\end{pmatrix}$