Problem 1 : Simplification

Simplify or compute the following expressions:

1. $2\begin{pmatrix}3\\4\end{pmatrix} - 2\begin{pmatrix}1\\-1\end{pmatrix}$; $\begin{pmatrix}1\\2\\3\end{pmatrix} \cdot \begin{pmatrix}-3\\1\\2\end{pmatrix}$; $\begin{pmatrix}1\\2\\3\end{pmatrix} \times \begin{pmatrix}-3\\1\\2\end{pmatrix}$
2. $\begin{pmatrix}3t\\4\end{pmatrix} + 2\begin{pmatrix}t+1\\t-1\end{pmatrix}$
3. $\frac{1}{2}\begin{pmatrix}1\\t\\t^2\end{pmatrix} + \frac{1}{t}\begin{pmatrix}t\\(t+1)^2\\\frac{2}{t}\end{pmatrix} + \frac{1}{t}\begin{pmatrix}t\\t^2\\t^3\end{pmatrix}$
4. $\begin{pmatrix}1\\\cos\theta\\\sin\theta\end{pmatrix} \cdot \begin{pmatrix}1\\\cos\theta\\\sin\theta\end{pmatrix}$
5. $\begin{pmatrix}1\\\cos\theta\\\sin\theta\end{pmatrix} \times \begin{pmatrix}t\\t\cos\theta\\t\sin\theta\end{pmatrix}$
6. $\left\| \begin{pmatrix}r\sin\theta\sin\omega\\r\sin\theta\cos\omega\\r\cos\theta\end{pmatrix} \right\|$

1. $\begin{pmatrix}4\\10\end{pmatrix}$; 5 ; $\begin{pmatrix}1\\-11\\7\end{pmatrix}$

2. $\begin{pmatrix}5t+2\\2t+2\end{pmatrix}$

3. $\begin{pmatrix}\frac{7}{2}\\\frac{7}{2}t + 2 + \frac{1}{t}\\\frac{3}{2}t^2 + \frac{2}{t^2}\end{pmatrix}$

4. 2

5. $\vec{0}$

6. $\sqrt{r^2\sin^2\theta\sin^2\omega + r^2\sin^2\theta\cos^2\omega + r^2\cos^2\theta} = \sqrt{r^2\sin^2\theta\cdot(\sin^2\omega+\cos^2\omega) + r^2\cos^2\theta}$

$= |r|$

$= \sqrt{r^2\sin^2\theta + r^2\cos^2\theta}$

Problem 2

Page 16, Problem 2: Determine whether the expressions are legal or not, and if legal, determine the expression is a vector or a number:

1. $(\vec{a}\cdot\vec{b})\times\vec{c}$; ✗
2. $(\vec{a}\times\vec{b})\cdot\vec{c}$; number
3. $\left\|\vec{a}\times\vec{b}\right\|$ number

P16. 2 v v v n n v

a) b) c) f) g) h) ✓

d) e) ✗

Problem 3: more simplification Page 17 Problem 13, 15

P17 13. a) $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b}$ ✓

$= \vec{a}\cdot\vec{a} - \vec{b}\cdot\vec{b} = \|\vec{a}\|^2 - \|\vec{b}\|^2$

b) $\|\vec{a}+\vec{b}\|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b}$ ✓

$= \|\vec{a}\|^2 + 0 + 0 + \|\vec{b}\|^2$

c) $\|\vec{a}-\vec{b}\|^2 = \|\vec{a}+(-\vec{b})\|^2 = \|\vec{a}\|^2 + \|-\vec{b}\|^2 = \|\vec{a}\|^2 + \|\vec{b}\|^2$

$\neq \|\vec{a}\|^2 - \|\vec{b}\|^2$ ✗

15. a) $\vec{0}$ b) $\vec{0}$

c) $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b}) = \vec{a}\times\vec{a} + \vec{a}\times\vec{b} - \vec{b}\times\vec{a} - \vec{b}\times\vec{b}$

$= 0 + \vec{a}\times\vec{b} + \vec{a}\times\vec{b} - 0 = 2\,\vec{a}\times\vec{b}$

d) $(\vec{a}+\vec{b}-\vec{c})\times(\vec{a}-\vec{b}+\vec{c}) = 2\cdot\vec{a}\times(\vec{b}-\vec{c}) = 2\vec{a}\times\vec{b} - 2\vec{a}\times\vec{c}$

e) $(\vec{a}+\vec{b}-\vec{c})\cdot(\vec{a}-\vec{b}+\vec{c}) = \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} + \vec{b}\cdot\vec{c}$

$- \vec{c}\cdot\vec{a} + \vec{c}\cdot\vec{b} - \vec{c}\cdot\vec{c}$

$= \|\vec{a}\|^2 - \|\vec{b}\|^2 - \|\vec{c}\|^2 + 2\,\vec{b}\cdot\vec{c}$