MATH 234 QUIZ 5 20 OCT 2016, TA: JIUYA WANG

Name:

Notice:
1. Please box your final answer.
2. Please stop writing when time is up.

Problem 1 (10 points):
Given the following surface:

$$xy + yz + xz = -1$$

1. Using implicit function theorem, if we can determine $z = f(x, y)$ to be implicit function depending on x and y, to compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$;
2. At which point(s) z is not an implicit function depending on x and y?
3. Find critical point(s) of function $z = f(x, y)$ you found in part 1.

1. $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{y+z}{x+y}$ $\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{x+z}{x+y}$

2. $F_z = 0 = x + y \Rightarrow -x^2 = -1$ $x = \pm 1$ so the pts are: $(1, -1, z)$ and $(-1, 1, z)$

z is arbitrary value

3. $\begin{cases} \frac{\partial z}{\partial x} = 0 \\ \frac{\partial z}{\partial y} = 0 \end{cases} \Rightarrow \begin{matrix} y + z = 0 \\ x + z = 0 \end{matrix} \Rightarrow x = y = -z$ so. $-x^2 = -1$ $x = \pm 1$.

the c.p are: $(1, 1, -1)$ and $(-1, -1, 1)$

Problem 2 (10 points):
Given

$$f(x, y) = e^{x+y^2}$$

and let

$$x(u, v) = u^2 - v^2, \; y(u, v) = 2uv$$

, consider the function $g(u, v) = f(x(u, v), y(u, v))$, compute $\frac{\partial g}{\partial u}$ and $\frac{\partial g}{\partial v}$.

$$\frac{\partial g}{\partial u} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial u} = e^{x+y^2} \cdot 2u + e^{x+y^2} \cdot 2y \cdot 2v$$

$$\frac{\partial g}{\partial v} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial v} = e^{x+y^2} \cdot (-2v) + e^{x+y^2} \cdot 2y \cdot 2u$$