Worksheet 9.

P1. 1. $\int_0^1 \int_0^4 x \, dy \, dx = \int_0^1 \left(\int_0^4 x \, dy \right) dx = \int_0^1 (x \cdot y) \Big|_{y=0}^{y=4} dx$

$= \int_0^1 4x \, dx = 2x^2 \Big|_0^1 = 2$

2. $\int_0^1 \int_0^4 x \cdot dx \, dy = \int_0^1 \left(\int_0^4 x \cdot dx \right) dy = \int_0^1 \left(\frac{x^2}{2} \Big|_{x=0}^{x=4} \right) dy$

$= \int_0^1 8 \, dy = 8y \Big|_0^1 = 8$

3. $\int_{-1}^1 \int_0^{x^2} (x^2 + y^2) \, dy \, dx = \int_{-1}^1 \left(x^2 \cdot y + \frac{y^3}{3} \right) \Big|_{y=0}^{y=x^2} dx$

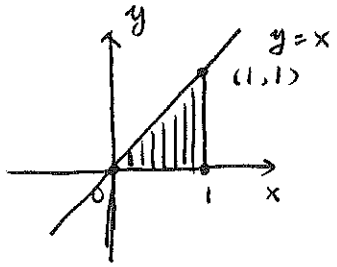
$= \int_{-1}^1 \left(x^4 + \frac{x^6}{3} \right) dx = \left(\frac{x^5}{5} + \frac{x^7}{21} \right) \Big|_{-1}^1 = \frac{52}{105}$

4. $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x \, dy \, dx = \int_0^1 (x \cdot y) \Big|_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} dx$

$= \int_0^1 2x \cdot \sqrt{1-x^2} \, dx \overset{u=1-x^2}{=} -\int_1^0 \sqrt{u} \, du = \frac{2}{3} \cdot u^{\frac{3}{2}} \Big|_0^1$

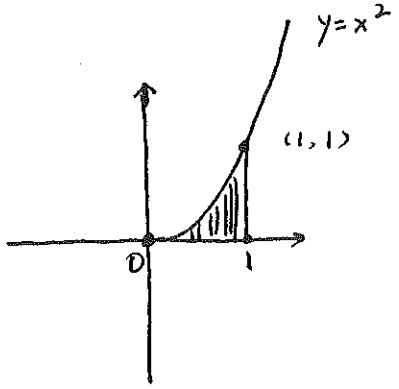
$= \frac{2}{3}$

P2. 1.



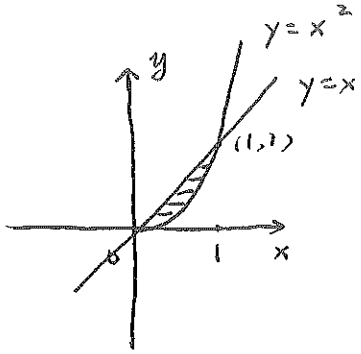
$\int_0^1 \int_y^1 f(x, y) \, dx \, dy$

2.



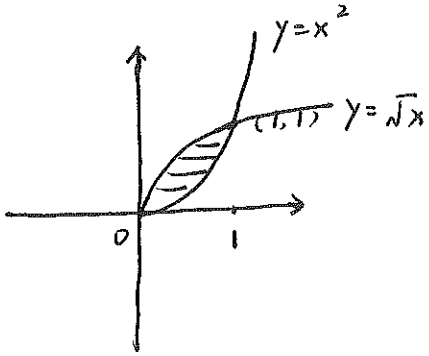
$\int_0^1 \int_{\sqrt{y}}^1 f(x, y) \, dx \, dy$

3.



$$\int_0^1 \int_y^{\sqrt{y}} f(x,y)\,dx\,dy$$

4.

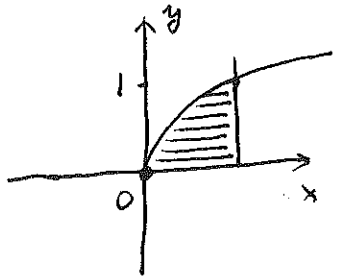


$$\int_0^1 \int_{y^2}^{\sqrt{y}} f(x,y)\,dx\,dy$$

P3. 1. $$\int_0^2 \int_{-x}^{x} (1+x)\,dy\,dx = \int_0^2 (y+xy)\Big|_{y=-x}^{y=x} dx = \int_0^2 (2x+2x^2)\,dx = \frac{28}{3}$$

2. $$\int_0^1 \int_0^x 1\cdot dy\,dx = \int_0^1 y\Big|_{y=0}^{y=x} dx = \int_0^1 x\,dx = \frac{1}{2}$$

3.

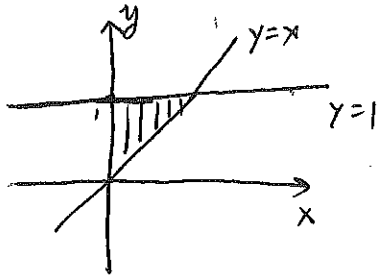


$$\int_0^1 \int_0^{\sqrt{x}} y\sin x^2\,dy\,dx$$

$$= \int_0^1 \sin x^2 \cdot \frac{y^2}{2}\Big|_0^{y=\sqrt{x}} dx = \int_0^1 \sin x^2 \cdot \frac{x}{2}\,dx$$

$$= \frac{1}{4}\cdot(-\cos x^2)\Big|_0^1 = \frac{1}{4}\cdot(1-\cos 1)$$

4.



$$\int_0^1 \int_x^1 \frac{2}{1-x^2}\,dy\,dx$$

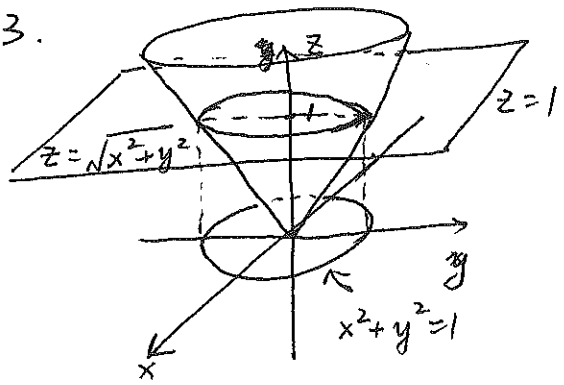
$$= \int_0^1 \frac{2}{1-x^2}\cdot(1-x)\cdot dx = \int_0^1 \frac{2}{x+1}\,dx$$

$$= 2\ln(x+1)\Big|_0^1 = 2\ln 2$$

P4. 1. $$\int_0^1 \int_0^{2\pi} r \cdot d\theta\, dr = \int_0^1 2\pi r \cdot dr = \pi r^2 \Big|_0^1 = \pi$$

2. $$\int_0^{\pi} \int_0^1 r^2 \cdot r\, dr\, d\theta = \int_0^{\pi} \frac{r^4}{4} \Big|_0^1 d\theta = \int_0^{\pi} \frac{1}{4} d\theta = \frac{\pi}{4}$$

3.



The volume is computed by taking the unit circle D as the basis. for each point in D, the corresponding height is $1 - z = 1 - r$

So $$V = \int_0^{2\pi} \int_0^1 (1-r) \cdot r\, dr\, d\theta = \int_0^{2\pi} \frac{1}{6} \cdot d\theta = \frac{\pi}{3}$$