

Name:

Notice:

1. Please box your final answer.
2. Please stop writing when time is up.

**Problem 1 (10 points):**

Given  $\vec{F} = \begin{pmatrix} x^2 - 2xy^3 \\ -3x^2y^2 \end{pmatrix}$ , and the curve  $C: \vec{\gamma}(t) = \begin{pmatrix} t^2 - t \\ \sin \pi t \end{pmatrix}$

1. Is  $\vec{F}$  conservative or not?
2. Consider the line integral of  $\vec{F}$  over  $\gamma$  from  $t = 0$  to  $t = 1$

1. Yes.  $Q_x = -6xy^2 = P_y = 0 - 6xy^2 = -6xy^2$

2.  $\vec{\gamma}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$   $\vec{\gamma}(1) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$   $\vec{F}$  is conservative.

so  $\oint_{\vec{\gamma}} \vec{F} \cdot d\vec{s} = 0$

**Problem 2 (10 points):**

Find the average  $x$  coordinates of the quarter of the unit circle that lies in the first quadrant ( $x \geq 0, y \leq 0$ ).

$\vec{\gamma}(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$   $0 \leq t \leq \frac{\pi}{2}$

Arc Length:  $\int_0^{\frac{\pi}{2}} \|\vec{\gamma}'(t)\| dt = \frac{\pi}{2}$

$x$  coordinates:  $\int_0^{\frac{\pi}{2}} \cos t \cdot \|\vec{\gamma}'(t)\| dt = \sin t \Big|_0^{\frac{\pi}{2}} = 1$

Ave:  $\frac{1}{\pi/2} = \frac{2}{\pi}$