MATH 234 QUIZ 10 8 DEC 2016, TA: JIUYA WANG

Name:

Notice:
1. Please box your final answer.
2. Please stop writing when time is up.

Problem 1 (10 points):
Compute the following integral by any method.

$$\int_C y^3dx + 2x^3dy \qquad \vec{F} = \begin{pmatrix} y^3 \\ 2x^3 \end{pmatrix}$$

where $C = \partial D$, $D : x^2 + y^2 \leq 4$ and C is clockwise.

$C: \vec{r}(t) = \begin{pmatrix} 2\cos t \\ 2\sin t \end{pmatrix}$ t is from 2π to 0

$$\int_{2\pi}^{0} \begin{pmatrix} 8\sin^3 t \\ 16\cos^3 t \end{pmatrix} \cdot \begin{pmatrix} -2\sin t \\ 2\cos t \end{pmatrix} dt$$

$$= \int_{2\pi}^{0} (-16\sin^4 t + 32\cos^4 t)\, dt$$

$$\int_0^{2\pi} \sin^4 t\, dt = \int_0^{2\pi} \frac{(1-\cos 2t)^2}{4} dt$$

$$= \frac{1}{4} \int_0^{2\pi} \left(1 - 2\cos 2t + \frac{1+\cos 4t}{2}\right) dt$$

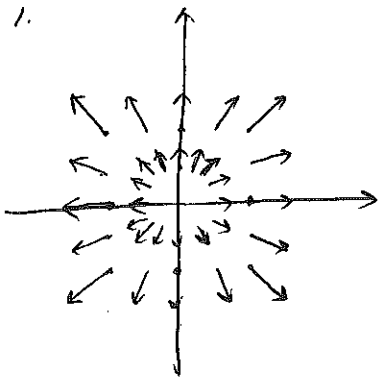
$$= \frac{1}{4} \cdot 3\pi$$

So the result is:

$$16 \cdot \frac{3\pi}{4} + 32 \cdot \left(-\frac{3\pi}{4}\right) = -12\pi$$

Problem 2 (10 points):
Water is flowing with velocity $\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$,
1. Draw the vector field;
2. How much water is flowing out the circle $x^2 + y^2 = 16$?

1.



$$2\, \mathrm{div}\vec{v} = P_x + Q_y = 2$$

$$\iint_D 2\, dA = 2 \cdot \iint_D 1\, dA$$

$$= 2 \cdot (16\pi) = 32\pi$$