Multi-variable Calculus

Calculus $<$ taking derivative / taking integral

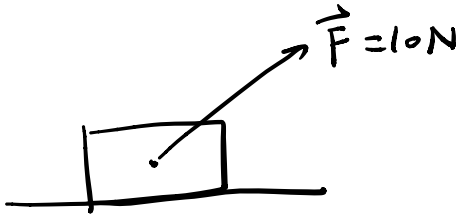
$y = f(x)$ $\quad f'(x) = \frac{df}{dx}$ $\quad \int f(x)\,dx$

$z = f(x,y)$ $\quad f(x_1, \cdots, x_n)$

Vectors.

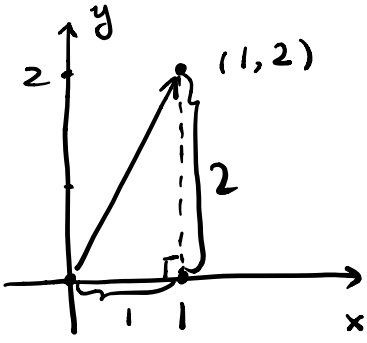
physics model : force , velocity.

What is vector ? vector "=" direction + scalar



$\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ← component

$= (1, 2)$



length / magnitude.

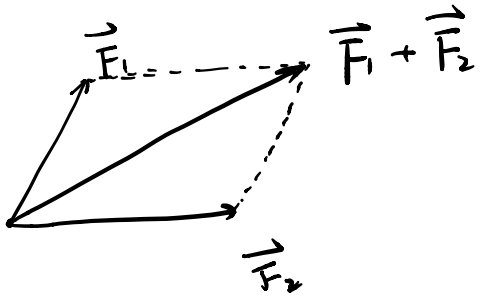
$\|\vec{v}\| = ?$ $\quad \sqrt{5}$

$= \sqrt{1^2 + 2^2}$

Operations of Vectors

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

- Addition

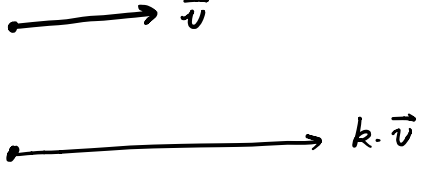


$$\vec{a} + \vec{b} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{pmatrix}$$

- Scalar Multiplication

preserve the direction

change the magnitude



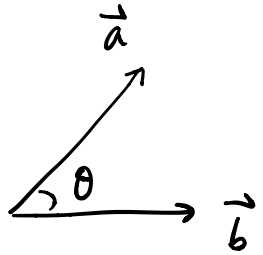
- Length of a vector :

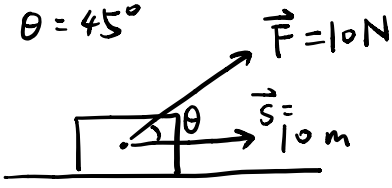
$$\| \vec{v} \| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

- Dot Product

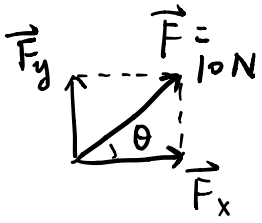
$$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3$$

$$= \| \vec{a} \| \cdot \| \vec{b} \| \cdot \cos\theta$$





$$W = \| \vec{F} \| \cdot \cos\theta \cdot \| \vec{s} \| = \vec{F} \cdot \vec{s}$$



$\rightarrow$

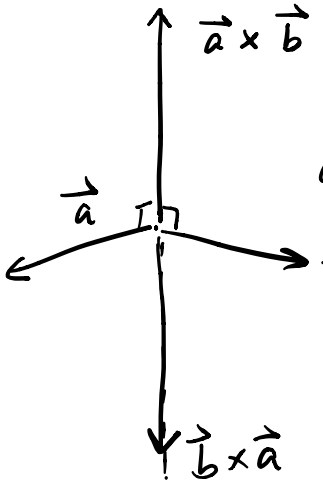
$\|\vec{F}_x\| = 10 \cdot \cos\theta$

$\vec{a} \cdot \vec{b} = 0 \iff \cos\theta = 0 \iff \vec{a} \perp \vec{b}$

- **Cross Product** (special for 3-dim)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

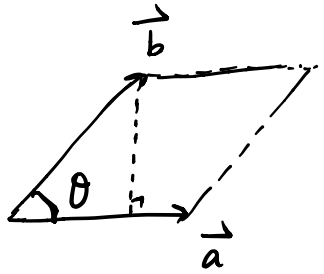
↖ determinant



direction: Right Hand Rule:

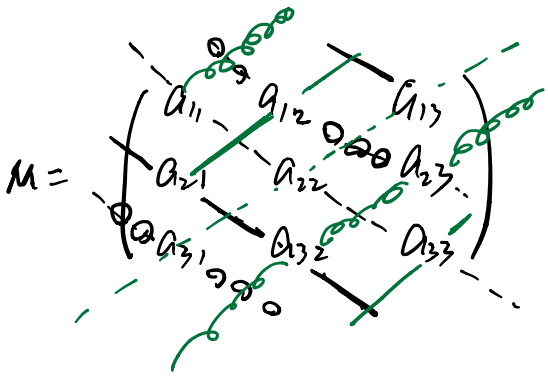
magnitude: $\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \sin\theta$

= Area ▱



$\vec{a} \times \vec{b} = \vec{0} \iff \sin\theta = 0 \iff \vec{a} \parallel \vec{b}$

$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$



$$\det(M) = \left| \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \right|$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$