- Grp Project (optional)
- Correction $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoid

  $\frac{x^2}{a^2}+\frac{y^2}{b^2}=z$ paraboloid.

Multivariable funtion (basic)

- D subset of $\mathbb{R}^n$

$$f: D \longrightarrow \mathbb{R}^m$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \quad \begin{pmatrix} y_1 = f_1(x_1, \cdots, x_n) \\ \vdots \\ y_m = f_m(x_1, \cdots, x_n) \end{pmatrix}$$

$$= f_m(\vec{x})$$

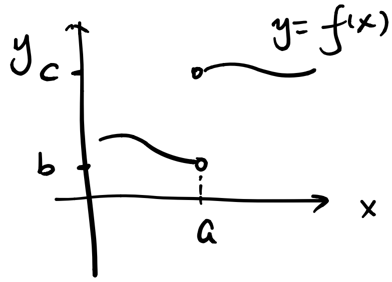
D: domain

- eg. $z = f(x, y)$ $\quad f: \mathbb{R}^2 \longrightarrow \mathbb{R}^1$

$$\begin{pmatrix} x \\ y \end{pmatrix} \quad z$$

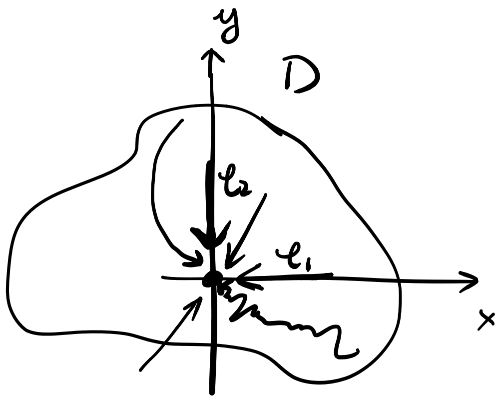
level set $\quad S(c) = \{ \vec{x} \in D \mid f(\vec{x}) = c \}$
for $c \in \mathbb{R}$

- limit



$\lim_{x \to a} f(x)$ DNE

$\lim_{x \to a^-} f(x) = b \neq \lim_{x \to a^+} f(x) = c$



limit exists only when the limits along all ways approaching the point are the same.

eg. $f(x,y) = \dfrac{x}{x+y}$

1) largest possible domain

$D = \{(x,y) \mid x+y \neq 0\}$

2) $\lim_{\vec{x} \to \vec{0}} f(x,y) = ?$ DNE

along x-axis

$f(x,y) = \dfrac{x}{x+0} = 1$ when $x \neq 0$.

along y-axis

$f(x,y) = \dfrac{0}{0+y} = 0$ when $y \neq 0$

$\lim_{\substack{\vec{x} \to \vec{0} \\ l_1}} f(x,y) = 1 \neq \lim_{\substack{\vec{x} \to \vec{0} \\ l_2}} f(x,y) = 0$

3) $\lim_{\vec{x} \to (1,0)} f(x,y) = ?$

$f(1,0) = \dfrac{1}{1+0} = 1$

- Continuous Function. $f(x)$ is continuous at $\vec{x} = \vec{a}$ if

$$\lim_{\vec{x} \to \vec{a}} f(\vec{x}) = f(\vec{a})$$

(*) poly / exp / ln / trig are continuous in their Domain.

(**) compositions of continuous functions are still continuous.

$\sin(e^x)$

- $\lim_{\vec{x} \to \vec{x}_0} f(\vec{x}) \begin{matrix} + \\ \times \\ - \\ \div \end{matrix} g(\vec{x}) = \lim_{\vec{x} \to \vec{x}_0} f(\vec{x}) \begin{matrix} + \\ \times \\ - \\ \div \end{matrix} \lim_{\vec{x} \to \vec{x}_0} g(\vec{x})$

as long as all limits mentioned exist

$\lim_{\vec{x} \to \vec{x}_0} f(g(\vec{x})) = \lim_{\vec{y} \to \vec{y}_0 = g(\vec{x}_0)} f(\vec{y})$ if g is continuous at $\vec{x}_0$.

Ex: 1) Find the longest possible domain

① $\frac{1}{x^2+y^2}$

$D = \mathbb{R}^2 \setminus (0,0)$

② $\ln(x+y)$

$D = \{ (x,y) \mid x+y>0 \}$

③ $\sqrt{x^2-y^2}$

$D = \{ (x,y) \mid |x| \geq |y| \}$

④ $\arctan(x+y)$

$D = \mathbb{R}^2$

⑤ $\frac{1}{\ln(x^2+y^2)}$

$D = \{ (x,y) \mid x^2+y^2 \neq 0, 1 \}$

⑥ $\ln \frac{1}{x-1}$

$D = \{ (x,y) \mid x-1>0 \}$

2) Prove the following limit DNE or compute the limit.

at $(0,0)$

① $\lim_{\vec{x}\to\vec{0}} x^2+y^2 = 0$

② $\lim_{\vec{x}\to\vec{0}} \frac{\sin(xy)}{xy} = 1$

$= \lim_{x\to 0} \frac{\sin(x)}{x} = 1$

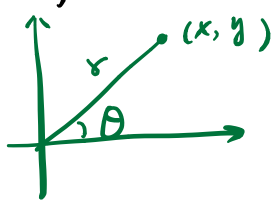
$f(\cdot) = \frac{\sin(\cdot)}{\cdot} \quad g(x,y) = x\cdot y$

③ $\lim_{\vec{x}\to\vec{0}} \frac{x^2+y^2}{x^2+2\cdot y^2}$ DNE

x-axis   y-axis

④ $\lim_{\vec{x}\to\vec{0}} \frac{x^3+y^3}{x^2+y^2} = 0$

pf:



$x = r\cos\theta$

$y = r\sin\theta$

$\frac{x^3+y^3}{x^2+y^2} = r\cdot(\cos^3\theta + \sin^3\theta)$

$\lim_{\vec{x}\to\vec{0}} f(x,y) = \lim_{r\to 0} r\cdot(\cos^3\theta+\sin^3\theta)$

$= 0$

⑤ $\lim_{\vec{x}\to\vec{0}} \frac{x^3+y^3}{x^2-y^2}$ DNE

pf x-axis 0 $\lim_{x\to 0} f(x,0)$

$x = y+y^2$ 1

$\frac{y^3+3\cdot y^4+3y^5+y^6+y^3}{\cancel{y^2}+2y^3+y^4-\cancel{y^2}}$

$= \frac{2+3y+3y^2+y^3}{2+y}$