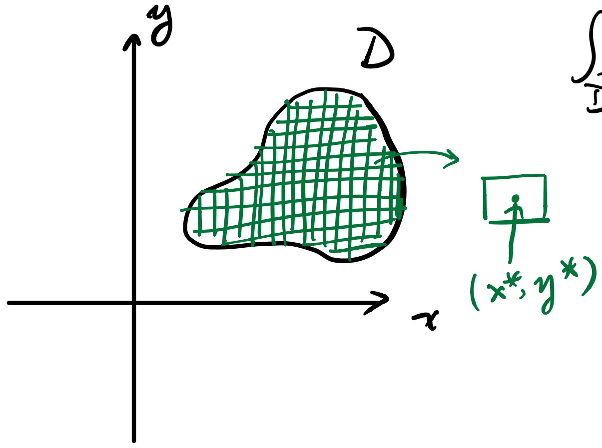
Double Integral: integral of 2-variable functions.



$$\iint_D f(x,y)\,.dA = \lim_{P} \sum_{\square} f(x^*,y^*).\text{Area}(\square)$$

Riemann. sum.

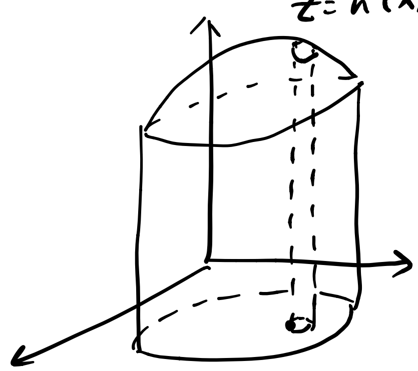
To compute:

1) Write a double integral into iterated integral.

2) Compute the iterated integral.

Application:

1) Volume.



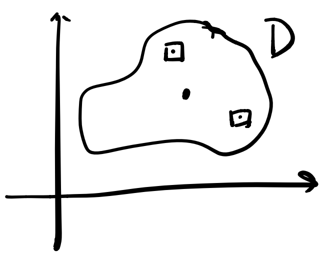
Area(□) · height. = volume of 

Add up. to get the total volume.

$$\text{volume} = \iint_D h(x,y)\,dA.$$

usually $h(x,y)$ need to be positive to actually mean a volume.

2) Mass. of a region.



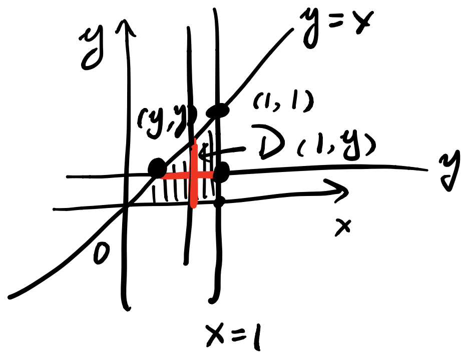
$d(x,y)$

$$\iint_D d(x,y)\,dA = \text{mass}$$

3) Center of a region.

$$\bar{x} = \frac{\iint_D x \cdot dA}{\iint_D 1\, dA} \qquad \bar{y} = \frac{\iint_D y\, dA}{\iint_D 1\, dA}.$$

eg. $\displaystyle\iint_D \overset{f(x,y)}{\overset{\shortparallel}{(x+y)}}\, dA$



① $\displaystyle\iint_D \quad dA.$

$$= \int_0^1 \left(\int_y^1 x+y. \quad dx \right) dy.$$

Find range of y-value.

Find range of x-value with a fixed y-value. you will get two functions in terms of outer variable.

do the integral as y is const

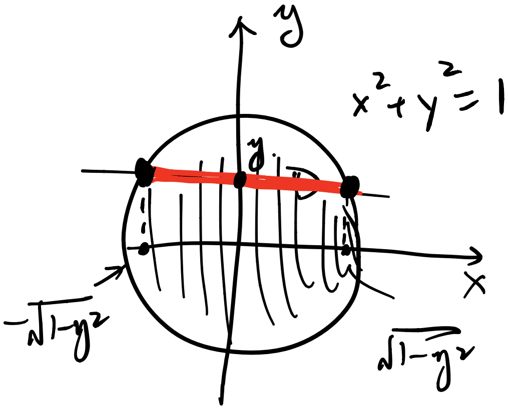
② $\displaystyle = \int_0^1 \left(\frac{x^2}{2} + y \cdot x \Big|_{x=y}^{x=1} \right) dy$

$$= \int_0^1 \left(\frac{1}{2} + y - \frac{y^2}{2} - y^2 \right) dy.$$

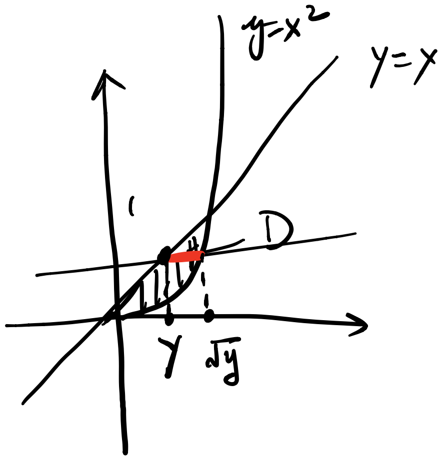
Ex. Try the same problem with. $\iint$ dy dx order.

Rmk: Step ① only cares about the region.

$$\int_0^1 \left(\int_0^x x+y\ dy \right) dx = \int_0^1 \left(\frac{y^2}{2} + yx \right) \Big|_{y=0}^{y=x} dx = \int_0^1 \frac{3}{2} x^2 \cdot dx.$$



$$\iint_D * \, dA = \int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} * \ dx\, dy$$



$$\iint_D x \, dA = \int_0^1 \int_y^{\sqrt{y}} x \, dx \, dy.$$