Triple Integral: Integral of 3-variable functions.

Input: region R 3-dim. Output: #. Riemann sum.

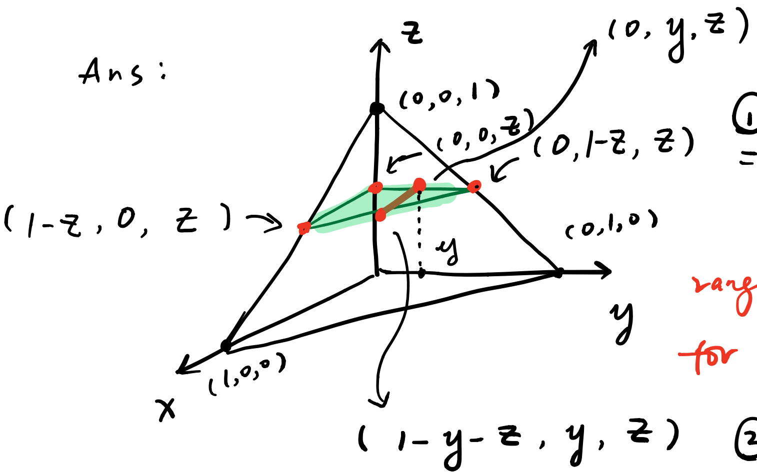
$f(x, y, z)$. $\iiint_R f(x,y,z)\cdot dV$.

- Rewrite triple integral to iterated integral
  ✱ only about R.
- Compute iterated integral.

eg. Region is bonded by $x+y+z=1$
$x=0$
$y=0$
$z=0$.

$f(x,y,z) = x\cdot y\cdot z$.

Ans:



$\iiint_R f(x,y,z)\cdot dV$

↙ range of y-values for all

① $= \int_0^1 \int_0^{1-z} \int_0^{1-y-z} x\cdot y\cdot z \; dx\, dy\, dz.$

← range of x-values for all

↗ range of z-values for all R.

② $= \int_0^1 \int_0^{1-z} \left. \frac{x^2}{2}\cdot y\cdot z \right|_{x=0}^{x=1-y-z} dy\,dz$

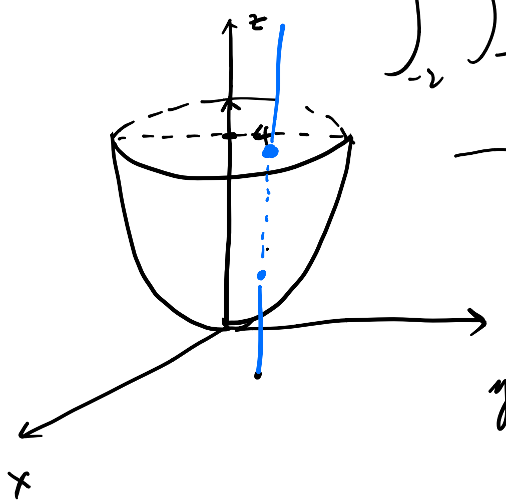
$= \int_0^1 \int_0^{1-z} \frac{(1-y-z)^2}{2}\, y\cdot z \; dy\,dz$

reduce to double integral iterated.

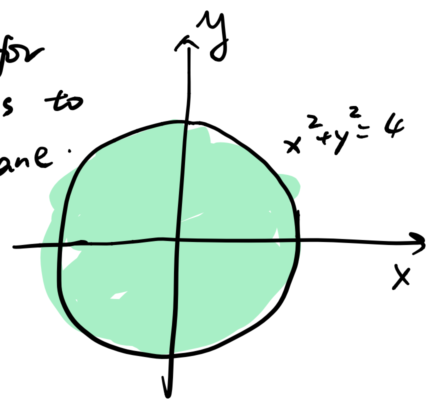
Ex. region is bonded by $z = x^2 + y^2$ and $z = 4$.

$$\iiint_R * \, dV = \int_{-2}^{2}\int_{-2}^{2}\int_{x^2+y^2}^{4} * \, dz\,dx\,dy$$

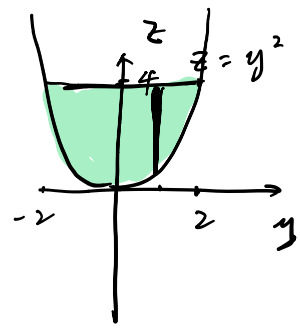
$$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{x^2+y^2}^{4} dz\,dx\,dy.$$



Look for projections to xy-plane.



$$\int_{-2}^{2}\int_{y^2}^{4}\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} dx\,dz\,dy$$



$$\int_{-2}^{2}\int_{x^2}^{4}\int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} dy\,dz\,dx$$