Divergence Thm / Stoke's Thm.

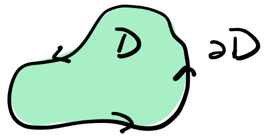
Recall Green's Thm

$(*)$ $\oint_{\partial D} \vec{F}\cdot\vec{T}\cdot ds = \iint_D (Q_x - P_y)\, dx\,dy$ (work)

∂D: simple closed

counter clockwisely

$(**)$ $\oint_{\partial D} \vec{F}\cdot\vec{N}\cdot ds = \iint_D (P_x + Q_y)\, dx\,dy$ (flux)



To generalize $(**)$ we replace $\oint_{\partial D}$ with $\oiint_S$

Divergence Thm:

$$\oiint_S \vec{F}\cdot\vec{N}\cdot ds = \iiint_T \underbrace{\vec{\nabla}\cdot\vec{F}}_{= P_x + Q_y + R_z}\, dx\,dy\,dz$$

S: closed surface

$\vec{N}$: outward normal

eg. S: unit sphere.

$\vec{F} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ what is the flux with outward normal?

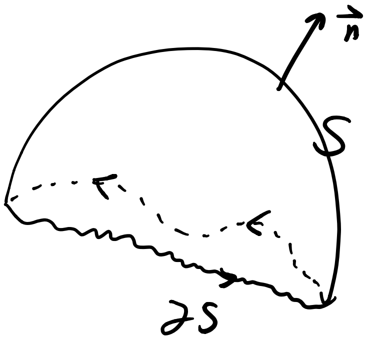
$$\oiint_S \vec{F}\cdot\vec{N}\, ds = \iiint_{\text{unit ball}} 3 \cdot dx\,dy\,dz = 3 \cdot \mathrm{Vol}(\text{unit ball})$$

$$= 3 \cdot \frac{4}{3}\pi = 4\pi$$

$\vec{F} = \begin{pmatrix} y \\ z \\ x \end{pmatrix}$ what is the flux? $0.$

because $\vec{\nabla}\cdot\vec{F} = 0$

To generalize (*) :



Stoke's Thm:

$$\oint_{\partial S} \vec{F}\cdot\vec{T}\,ds = \iint_S (\text{curl}\,\vec{F})\cdot\vec{N}\,dA$$

$$\shortparallel$$

$$\oint_{\partial S} \vec{F}\cdot d\vec{\gamma}$$

Orientation: use right hand to determine orientation

Thumb $\vec{N}$ for S

4 Fingers $\vec{T}$ for $\vec{\gamma}$

$$\oint_{\partial D} \vec{F}\cdot\vec{T}\cdot ds = \iint_D (Q_x - P_y)\,dx\,dy$$

(work)

$$\vec{F}' = \begin{pmatrix} P \\ Q \\ 0 \end{pmatrix} \qquad \text{curl}\,\vec{F}' = \begin{vmatrix} i & j & k \\ \partial x & \partial y & \cancel{\partial z} \\ P & Q & 0 \end{vmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ Q_x - P_y \end{pmatrix}$$

$$\text{curl}\,\vec{F}'\cdot\vec{N} = Q_x - P_y$$