Math 212 Worksheet 1 Aug 20 2020, Instructor: Jiuya Wang

Problem 1 : Vector computation

1. $2\begin{pmatrix}3\\4\end{pmatrix} - 2\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}4\\10\end{pmatrix}$

2. $\begin{pmatrix}1\\2\\3\end{pmatrix} \cdot \begin{pmatrix}-3\\1\\2\end{pmatrix} = 5$

3. $\begin{pmatrix}1\\2\\3\end{pmatrix} \times \begin{pmatrix}-3\\1\\2\end{pmatrix} = \begin{pmatrix}1\\-11\\7\end{pmatrix}$ $\qquad \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 2 & 3\\ -3 & 1 & 2\end{vmatrix} = \begin{pmatrix}1\\-11\\7\end{pmatrix}$

4. $\left\|\begin{pmatrix}1\\2\\3\end{pmatrix}\right\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}.$

Problem 2 : Simplification

Simplify or compute the following expressions:

1. $\begin{pmatrix}3t\\4\end{pmatrix} + 2\begin{pmatrix}t+1\\t-1\end{pmatrix} = \begin{pmatrix}5t+2\\2t+2\end{pmatrix}$

2. $\begin{pmatrix}1\\\cos\theta\\\sin\theta\end{pmatrix} \cdot \begin{pmatrix}1\\\cos\theta\\\sin\theta\end{pmatrix} = 2$ $\qquad \sin^2\theta + \cos^2\theta = 1$

3. $\begin{pmatrix}1\\\cos\theta\\\sin\theta\end{pmatrix} \times \begin{pmatrix}t\\t\cos\theta\\t\sin\theta\end{pmatrix} = \vec{0}$

4. $\left\|\begin{pmatrix}r\sin\theta\sin\omega\\r\sin\theta\cos\omega\\r\cos\theta\end{pmatrix}\right\| = \sqrt{\underbrace{r^2\sin^2\theta\sin^2\omega + r^2\sin^2\theta\cdot\cos^2\omega}_{} + r^2\cos^2\theta}$

$= |r|$

$= r^2\sin^2\theta\cdot(\sin^2\omega+\cos^2\omega)$

$= r^2\sin^2\theta\cdot 1$

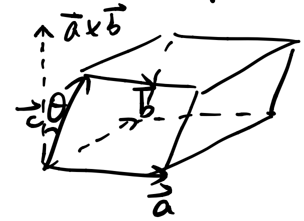
$+ \; r^2(\sin^2\theta+\cos^2\theta)$

$= r^2\cdot 1$

Problem 3

Page 16, Problem 2: Determine whether the expressions are legal or not, and if legal, determine the expression is a vector or a number:

1. $(\vec{a} \cdot \vec{b}) \times \vec{c}$;   X
2. $(\vec{a} \times \vec{b}) \cdot \vec{c}$;   ✓ |Triple product| = volume of cube · gen by $\vec{a}, \vec{b}, \vec{c}$
3. $\left\| \vec{a} \times \vec{b} \right\|$ ✓



$B = \|\vec{a} \times \vec{b}\|$

$H = \|\vec{c}\| \cdot \cos\theta$

Problem 4: Geometry and Vectors

1. Given $A(1, c)$, $B(2, 3)$ and $O(0, 0)$. For what value of c we have $\vec{OA} \perp \vec{OB}$? For what value of c we have $\vec{OA}$ is parallel to $\vec{OB}$?
2. l is the plane passing through $A(1, 2, 0)$, $B(0, 0, 1)$, $C(0, 1, 0)$.
   (a) What is the normal vector $\vec{n}$ of l?
   (b) Given arbitrary point X: (x, y, z) on the plane, what is $\vec{AX} \cdot \vec{n}$?
   (c) What is the defining equation of l?

$\|\vec{PQ} \times \vec{RQ}\| \cdot \frac{1}{2}$

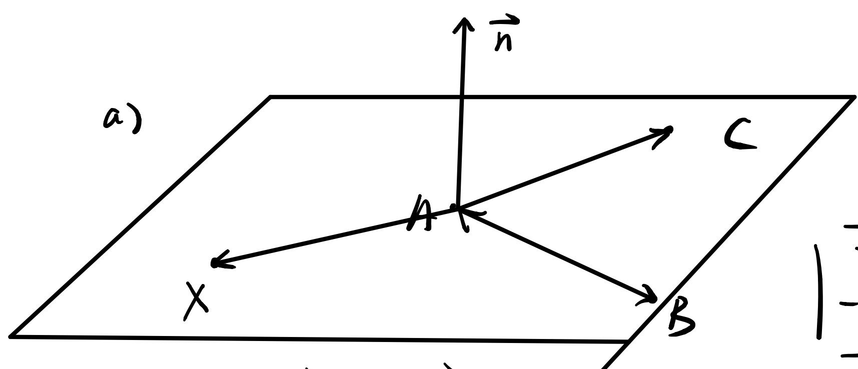
3. Find the area of the triangle with vertices $P(1, 1, 0)$, $Q(1, 0, 1)$ and $R(-3, -2, 2)$.
4. Find the volume of the polygon with vertices $O(0, 0, 0)$, $P(1, 1, 0)$, $Q(1, 0, 0)$ and $R(1, 1, 1)$.

1. $c = -\frac{2}{3}$ $\begin{pmatrix} 1 \\ c \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 2 + 3c = 0 \Rightarrow c = \frac{-2}{3}$

$c = \frac{3}{2}$ $\vec{OA} \parallel \vec{OB} \Leftrightarrow \begin{pmatrix} 1 \\ c \end{pmatrix} = t \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $2t = 1 \Rightarrow t = \frac{1}{2}$

$c = 3t = \frac{3}{2}$.

2. a)



$\vec{AB} \times \vec{AC} = \vec{n}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -2 & 1 \\ -1 & -1 & 0 \end{vmatrix} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

$\vec{BA} = -\vec{AB}$

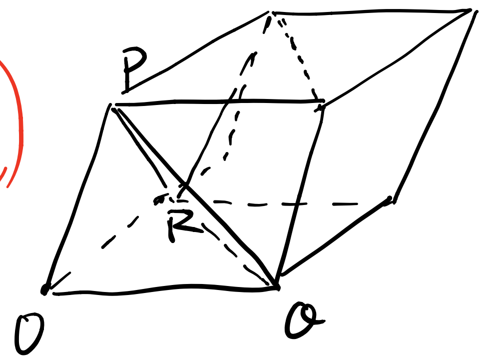
b) $\vec{AX} \cdot \vec{n} = 0$

c) $X: (x, y, z)$

$$\vec{AX} \cdot \vec{n} = \begin{pmatrix} x-1 \\ y-2 \\ z-0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = (x-1) - (y-2) - (z-0) = 0$$

$\Leftrightarrow x - y - z + 1 = 0$

3. $(\vec{OP} \times \vec{OQ}) \cdot \vec{OR} \cdot \frac{1}{6}$



Be careful of the extra factor $\frac{1}{2}$, $\frac{1}{6}$.