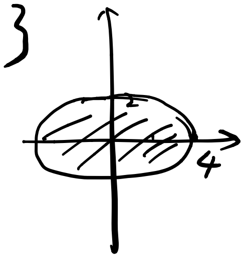
Problem 1: Domain

Find the largest domain where the functions can be defined and draw the region:

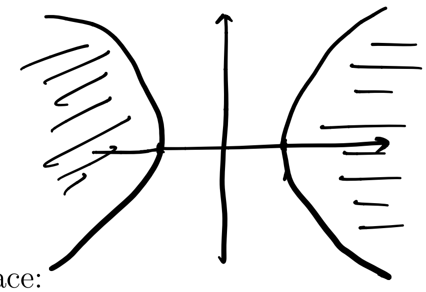
1. $f(x,y) = \sqrt{9-x^2} + \sqrt{y^2-4}$
2. $f(x,y) = \frac{1}{\sqrt{16-x^2-4y^2}}$
3. $f(x,y) = \sqrt{x^2+2xy-3y^2}$
4. $f(x,y) = \ln(x^2-y^2-1)$
5. $f(x,y,z) = \ln(4-x^2-y^2-z^2)$
6. $f(x,y,z) = \sqrt{z^2-x^2-y^2}$

2. $D = \{ (x,y) \mid x^2 + 4y^2 < 16 \}$

$\frac{x^2}{4^2} + \frac{y^2}{2^2} < 1$



4. $D = \{ (x,y) \mid x^2 - y^2 > 1 \}$.

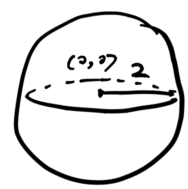


Problem 2: Level Set

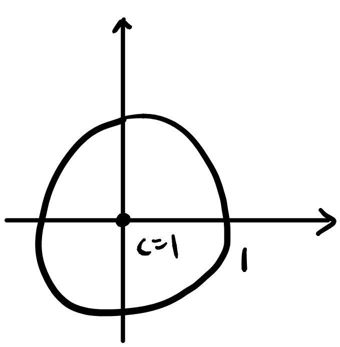
Determine the following level set and draw the level curve/surface:

1. $f(x,y) = x^2+y^2+1$, $c=1$, $c=2$
2. $f(x,y) = \sqrt{x-y}$, $c=1$, $c=2$.
3. $f(x,y) = \sqrt{x^2+2xy-3y^2}$, $c=0$
4. $f(x,y,z) = (x-1)^2+(y-2)^2+(z-1)^2/4$, $c=4$.
5. Textbook Chap 13.2, 53-58

5. $D = \{ (x,y,z) \mid x^2+y^2+z^2 < 4 \}$



1.



$x^2+y^2+1 = 1$

$x^2+y^2 = 0$

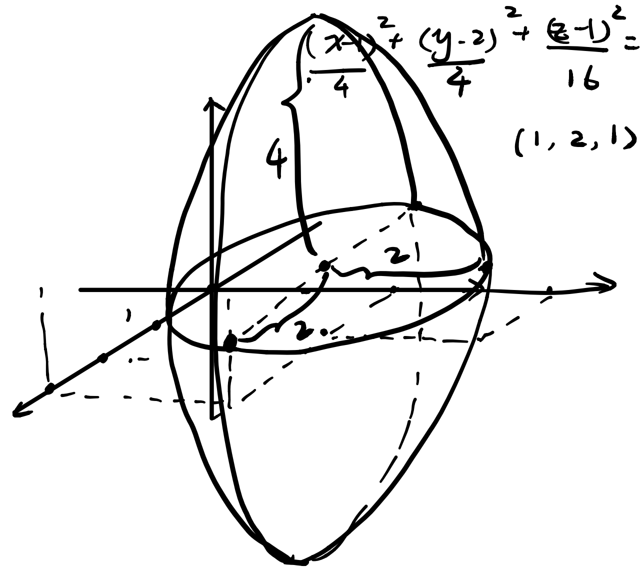
$x^2+y^2+1 = 2$

$x^2+y^2 = 1$

4.

$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{4} + \frac{(z-1)^2}{16} = 1$

$(1,2,1)$



Problem 3: Limit

Determine the following limit:

1. $f(x,y) = \frac{x^3-3y^3}{x^3+y^3}$, $a = (0,0)$
2. $f(x,y) = \frac{1-\cos(\sqrt{x^2+y^2})}{x^2+y^2}$, $a = (0,0)$ $\frac{1}{2}$
3. $f(x,y) = \sqrt{x^2+2xy-3y^2}$, $a = (0,0)$
4. $f(x,y) = \frac{1}{\ln(x^2+y^2)}$, $a = (0,0)$ $= 0$
5. $f(x,y,z) = \frac{x^2+2y^2+3z^2}{y^2+z^2+x^2}$, $a = (0,0,0)$ DNE
6. $f(x,y) = \frac{x^3-y^4}{x^2+y^2}$, $a = (0,0)$.

$\frac{\sin(xy)}{xy}$

1. DNE $y = kx$

3. 0 continuous at 0

6. 0 $x = r \cdot \cos\theta$

$y = r \cdot \sin\theta$

$$f(x,y) = \frac{r^3\cos^3\theta - r^4\sin^4\theta}{r^2}$$

$$= r \cdot \cos^3\theta - r^2 \cdot \sin^4\theta$$

$$\lim_{r\to 0}(r\cos^3\theta - r^2\cos^4\theta) = 0$$

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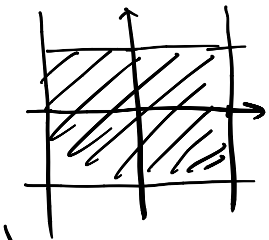
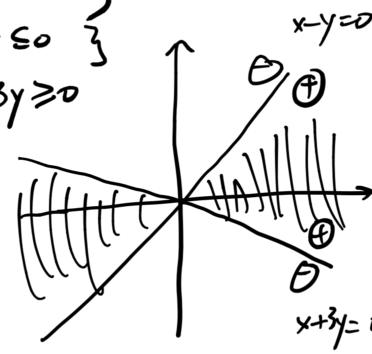
1. $D = \{(x,y) \mid |x| \le 3,\ |y| \le 2\}$

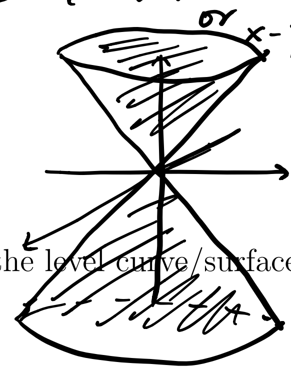


2. $f(x,y) = \frac{1}{\sqrt{16-x^2-4y^2}}$
3. $f(x,y) = \sqrt{x^2+2xy-3y^2}$ (Hint: factorize the polynomial)
4. $f(x,y) = \ln(x^2-y^2-1)$
5. $f(x,y,z) = \ln(4-x^2-y^2-z^2)$
6. $f(x,y,z) = \sqrt{z^2-x^2-y^2}$

3. $D = \{(x,y) \mid x-y \le 0\ \ x+3y \le 0\}$ or $x-y \ge 0\ \ x+3y \ge 0$



6. $D = \{(x,y) \mid z^2 \ge x^2+y^2\}$



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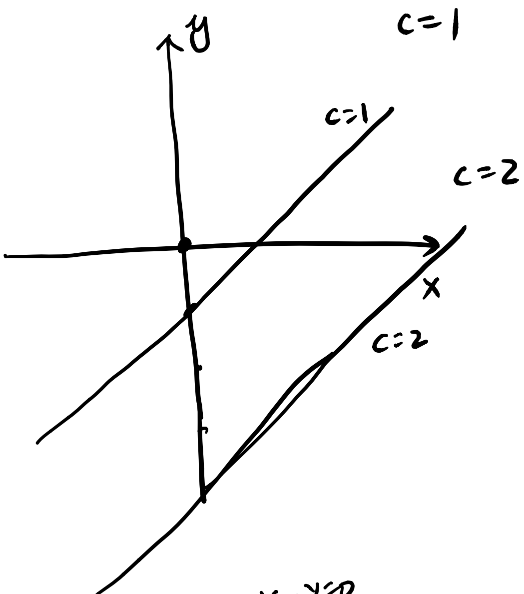
2.



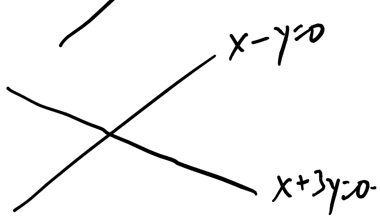
$c=1$ $\sqrt{x-y}=1$

$x-y=1$

$c=2$ $x-y=4$

5

3.



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2. $f(x,y) = \frac{1-\cos(\sqrt{x^2+y^2})}{x^2+y^2}$, $a = (0,0)$ $= \frac{1}{2}$

   composition of continuous func.

   $\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$.
3. $f(x,y) = \sqrt{x^2+2xy-3y^2}$, $a = (0,0)$
4. $f(x,y) = \frac{1}{\ln(x^2+y^2)}$, $a = (0,0)$ $= 0$
5. $f(x,y,z) = \frac{x^2+2y^2+3z^2}{y^2+z^2+x^2}$, $a = (0,0,0)$ DNE

   $\vec{r}(t) = \begin{pmatrix} at \\ bt \\ ct \end{pmatrix}$
6. $f(x,y) = \frac{x^3-y^4}{x^2+y^2}$, $a = (0,0)$.