MATH 212 WORKSHEET 5 24 SEPT 2020, INSTRUCTOR: JIUYA WANG

Problem 1 : Iterated Integral
Compute the following iterated integral:

1. $\int_0^1 \int_0^4 x dy dx$; $\int_0^1 \int_0^4 x dx dy$;
2. $\int_{-1}^1 \int_0^{x^2} (x^2 + y^2) dy dx$;
3. $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x dy dx$;

1. $\int_0^1 (x\cdot y)\Big|_{y=0}^{y=4} dx = \int_0^1 4x\cdot dx = 2x^2\Big|_{x=0}^{x=1} = 2$

$\int_0^1 \frac{x^2}{2}\Big|_{x=0}^{x=4} dy. = \int_0^1 8. dy = 8.$

3. $\int_0^1 2x\cdot\sqrt{1-x^2}\,dx = \int_0^1 \sqrt{1-x^2}\,.dx^2$

$= -\frac{(1-x^2)^{3/2}}{3/2}.\Big|_{x=0}^{x=1}$

$= \frac{2}{3}$

2. $\int_{-1}^1 (x^2\cdot x^2 + \frac{x^6}{3})\, dx$

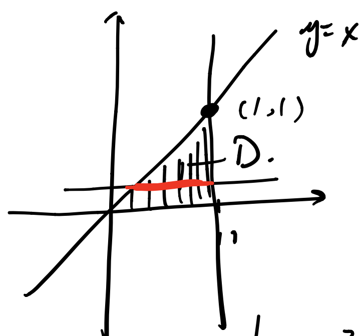
$= (\frac{x^5}{5} + \frac{x^7}{21})\Big|_{-1}^1$

$= \frac{2}{5} + \frac{2}{21}$

Problem 2 : Domain is important!
For all the following iterated integral:
1. Draw the domain D for the corresponding double integral;
2. Rewrite your double integral back to iterated integral with the other order.

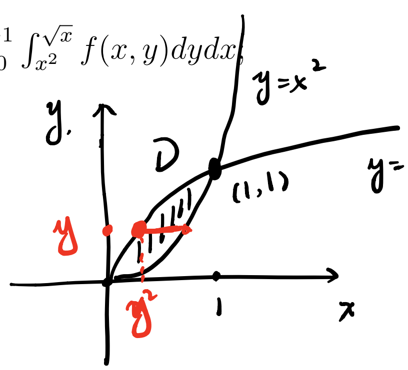
1. $\int_0^1 \int_0^x f(x, y) dy dx$;
2. $\int_0^1 \int_0^{x^2} f(x, y) dy dx$;
3. $\int_0^1 \int_{x^2}^x f(x, y) dy dx$;
4. $\int_0^1 \int_{x^2}^{\sqrt{x}} f(x, y) dy dx$;

1.



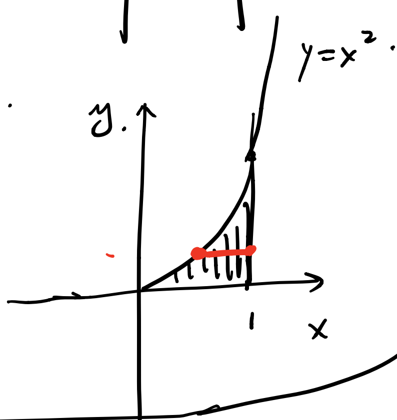
$\int_0^1 \int_y^1 f\,.\,dx\,dy.$

4.



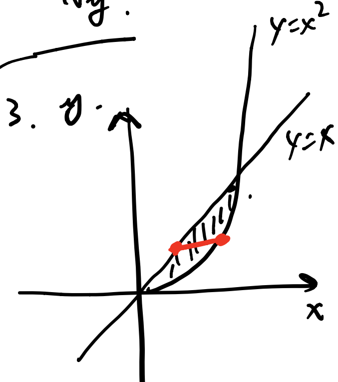
$\int_0^1 \int_{y^2}^{\sqrt{y}} f\ dx\,dy$

2.



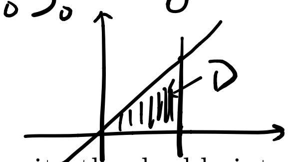
$\int_0^1 \int_{\sqrt{y}}^1 f\ dx\,dy$

3.



$\int_0^1 \int_y^{\sqrt{y}} f\ dx\,dy$

2. $\int_0^1\int_0^x 1\, dy\, dx = \int_0^1 x\cdot dx = \frac{1}{2} = \text{Area}(D))$

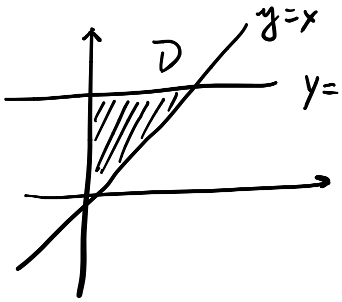


Problem 3 : Double Integral

For the following double integral:
1. Draw the domain and pick one order to write the double integral to iterated integral;
2. Compute the iterated integral;

1. $\iint_D (1+x)dA$; $D = \{(x,y) : 0 \le x \le 2, -x \le y \le x\}$
2. $\iint_D dA$; $D = \{(x,y) : 0 \le x \le 1, 0 \le y \le x\}$
   (Q: what are you computing when the integrand is 1 ?)
3. $\iint_D y\sin(x^2)dA$; $D = \{(x,y) : 0 \le y \le 1, y^2 \le x \le 1\}$
   Hint: can you integrate? How about changing the order as you did in Problem 2?
4. $\iint_D \frac{2}{1-x^2}dA$; D is the triangle bounded by the y axis, $y = 1$, and $y = x$

4.



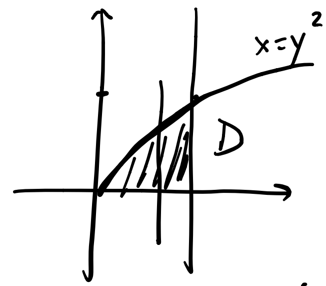
$\int_0^1\int_x^1 \frac{2}{1-x^2}\, dy\, dx$

$= \int_0^1 \frac{2}{1-x^2}\cdot(1-x)\, dx$

$= \int_0^1 \frac{2}{1+x}\, dx = 2\cdot\ln|x+1|\Big|_0^{x=1}$

$= 2\cdot\ln 2$

3.



If $\int_0^1\int_{y^2}^1 y\cdot\sin(x^2)\, dx\, dy$

$= \int_0^1\int_0^{\sqrt{x}} y\cdot\sin(x^2)\, dy\, dx = \int_0^1 \frac{x\cdot\sin x^2}{2}\cdot dx$ $u=x^2$

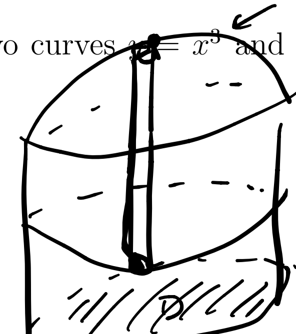
$= -\frac{\cos x^2}{2}\Big|_0^1$

$\frac{1-\cos 1}{4}$ =

Problem 4 : Application

1. Find the volume of the region between the two surfaces $z = x + y$ and $z = -x^2 - y^2$ over $D = \{(x,y) : 1 \le x \le y, 1 \le y \le 2\}$.
2. Find the mass of a metal disk with shape D cut out by two curves $y = x^3$ and $y = x$ where the density function is $d(x,y) = x^2 + y^2$.

$z = f_1(x,y)$



$z = f_2(x,y)$

$\iint_D (f_1 - f_2)\, dA$

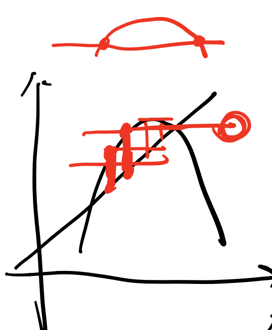
1. $\text{Vol} = \iint_D (x+y) - (-x^2 - y^2)\, dA$

$= \int_1^2\int_1^y x + x^2 + y + y^2\, dx\, dy$

$= \int_1^2 (y+y^2)\cdot(y-1) + (\frac{y^2}{2} + \frac{y^3}{3}) - (\frac{1}{2} + \frac{1}{3})\, dy$

$= \int_1^2 (\frac{4}{3}y^3 + \frac{y^2}{2} - y - \frac{5}{6})\, dy$

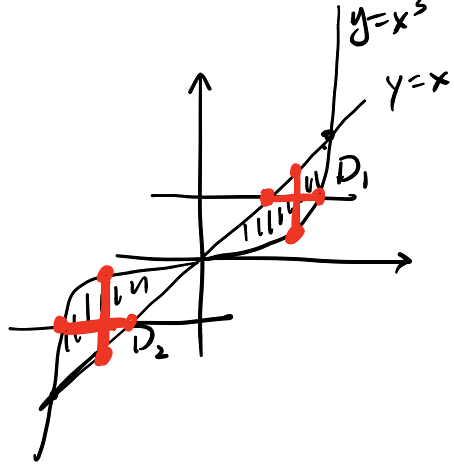
$= (\frac{y^4}{3} + \frac{y^3}{6} - \frac{y^2}{2} - \frac{5}{6}y)\Big|_1^2$



dx dy

dy dx

2.



mass $= \iint d(x,y)\, dA$

$D = D_1 \cup D_2$

For D_1

$\iint_{D_1} d(x,y)\, dA = \int_0^1 \int_y^{y^{1/3}} (x^2+y^2)\, dx\, dy$

$= \int_0^1 \left(\frac{x^3}{3} + y^2 \cdot x \right) \Big|_{x=y}^{x=y^{1/3}} dy.$

$= \int_0^1 \left(\frac{y}{3} - \frac{y^3}{3} + y^{\frac{7}{3}} - y^3 \right) dy$

$= \frac{y^2}{6} - \frac{y^4}{12} + \frac{3 \cdot y^{\frac{10}{3}}}{10} - \frac{y^4}{4} \Big|_0^1$

over.
D_1

$\int_0^1 \int_{x^3}^{x} (x^2+y^2)\, dy\, dx$

$= \int_0^1 x^2 \cdot (x - x^3) + \frac{x^3 - x^9}{3}\, dx$

$= \int_0^1 \left(\frac{4}{3} \cdot x^3 - x^5 - \frac{x^9}{3} \right) dx$

$= \frac{x^4}{3} - \frac{x^6}{6} - \frac{x^{10}}{30} \Big|_0^1 = \frac{1}{3} - \frac{1}{6} - \frac{1}{30}$

$= \frac{10-5-1}{30} = \frac{4}{30} = \frac{2}{15}$

over D : 4/15.