Problem 1 : Conservative Vector Field

1. Given $\vec{F} = \begin{pmatrix} 2xe^{xy} + x^2ye^{xy} \\ x^3e^{xy} + 2y \end{pmatrix}$, is $\vec{F}$ conservative or not? If so, find the potential function.
2. Given $\vec{F} = \begin{pmatrix} y \\ z \\ x \end{pmatrix}$, is $\vec{F}$ conservative or not?
3. Consider the vector field in 1.1, C is the upper half unit circle starting from $(-1, 0)$ to $(1, 0)$, compute the line integral of vector field.
4. Given a vector field $\vec{F} = (P(x, y), Q(x, y))$ on $\mathbb{R}^2$. Prove that $\vec{F}$ is conservative if and only if $P_y = Q_x$ if and only if the line integral of $\vec{F}$ does not depend on the path if and only if the line integral of $\vec{F}$ over a closed curve is 0.

Problem 2 : Green Theorem

Compute the following line integral over ∂D in two ways: by definition and by Green's Theorem:

1. $\vec{F} = \begin{pmatrix} 2xe^{xy} + x^2ye^{xy} \\ x^3e^{xy} + 2y \end{pmatrix}$, $D = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$
2. $\vec{F} = \begin{pmatrix} y\cos x \\ y\sin x \end{pmatrix}$, $D = \{(x, y) \mid 0 \leq \pi/2, 1 \leq y \leq 2\}$
3. $\vec{F} = \begin{pmatrix} x\sqrt{y} \\ \sqrt{x+y} \end{pmatrix}$, $D = \{(x, y) \mid 1 \leq x \leq 2, 2x \leq y \leq 4\}$

Problem 3: Flux Integral
Compute the following line integral:

1. $\vec{v} = \begin{pmatrix} x+y \\ 2y \end{pmatrix}$,$C : \vec{\gamma}(t) = (t, t^2), 0 \leq t \leq 1$, $\vec{N}$ the upward normal
2. $\vec{v} = \begin{pmatrix} xy^2 \\ x^2y \end{pmatrix}$, C: unit circle, $\vec{N}$ the outward normal