Prob 2. #4.

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} \times 10^{-1} \qquad \vec{x}_0 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$10 A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} = B$$

$$\lambda_i(A) \overset{?}{=} 10 \cdot \lambda_i(B)$$

$$\overset{?}{=} 10^{-1} \cdot \lambda_i(B)$$

$$\det(\lambda - A) = \det(\lambda - 10^{-1} B) \qquad \tilde{\lambda} = 10\lambda$$

$$= \det(10^{-1} \tilde{\lambda} - 10^{-1} B)$$

$$= (10^{-1})^n \cdot \det(\tilde{\lambda} - B) = 0$$

Suppose a is a root for $\det(\tilde{\lambda} - B) = 0$.

then. $10\lambda = a \iff \lambda = 10^{-1} \cdot a$ is a root for $\det(\lambda - A)$

$$\boxed{\begin{array}{l} B \vec{v}_i = \tilde{\lambda}_i \vec{v}_i \\ \underbrace{10^{-1} \cdot B}_{= A} \cdot \underset{\substack{\uparrow \\ \text{e.v.}}}{\vec{v}_i} = \underbrace{10^{-1} \cdot \tilde{\lambda}_i}_{\substack{\uparrow \\ \text{e.value}}} \underset{\substack{\uparrow \\ \text{e.v}}}{\vec{v}_i} \end{array}}$$

$$\vec{x}_0 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = a_1 \cdot \vec{v}_1 + a_2 \vec{v}_2 + a_3 \vec{v}_3$$

$$B = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

1)

$$\det(B-\lambda I) = \begin{vmatrix} -\lambda & 1 & 2 \\ 1 & -\lambda & 1 \\ 2 & 1 & -\lambda \end{vmatrix} = -\lambda^3 + 2 + 2 - (-\lambda) - (-\lambda) - 4\cdot(-\lambda)$$

$$= -\lambda^3 + 6\lambda + 4 \qquad \lambda_1 = -2 \quad \lambda_2 = \sqrt{3}+1$$

$$= (\lambda+2)\cdot(-(\lambda-1)^2 + 3) \qquad \lambda_3 = -\sqrt{3}+1$$

$$\begin{array}{r} -\lambda^2+2\lambda+2 \\ \lambda+2 \overline{\big) -\lambda^3 \quad +6\lambda+4} \\ -\lambda^3-2\lambda^2 \\ \hline 2\lambda^2+6\lambda+4 \\ 2\lambda^2+4\lambda \\ \hline 2\lambda+4 \end{array}$$

Therefore eigenvalues for A is

$-0.2 \quad \dfrac{\sqrt{3}+1}{10} \quad \dfrac{-\sqrt{3}+1}{10}$

and all of $|10^{-1}\lambda_i|$ are bonded by 1, therefore for any $\vec{x_0}$

$A^k\cdot\vec{x_0}$ will converge to $\vec{0}$.

$$\lim_{k\to\infty} A^k\cdot(a_1\vec{v_1} + a_2\vec{v_2} + a_3\vec{v_3}) = \lim_{k\to\infty} a_1\cdot A^k\cdot\vec{v_1} + a_2\cdot A^k\cdot\vec{v_2} + a_3 A^k\cdot\vec{v_3}$$

$$= \lim_{k\to\infty} a_1\cdot(\lambda_1 10^{-1})^k\vec{v_1} + a_2\cdot(\lambda_2\cdot 10^{-1})^k\vec{v_2} + a_3\cdot(\lambda_3\cdot 10^{-1})^k\vec{v_3} = \vec{0}.$$

Prob 5. #2.

$$f(x,y) = 2x^2 + 3xy + y^2$$

$$B = \begin{pmatrix} 2 & 3/2 \\ 3/2 & 1 \end{pmatrix}$$

$\det(B) < 0 \Rightarrow$ indefinite.

eigenvalue: $\det(\lambda I - B) = \begin{vmatrix} \lambda - 2 & -3/2 \\ -3/2 & \lambda - 1 \end{vmatrix}$

$$= (\lambda-1)(\lambda-2) - \frac{9}{4}$$

$$= \lambda^2 - 3\lambda + 2 - \frac{9}{4}$$

$$= \lambda^2 - 3\lambda - \frac{1}{4}$$

$$= \left(\lambda - \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{1}{4} = \left(\lambda - \frac{3}{2}\right)^2 - \frac{5}{2}$$

$$\lambda_1 = \sqrt{\frac{5}{2}} + \frac{3}{2} > 0$$

$$\lambda_2 = -\sqrt{\frac{5}{2}} + \frac{3}{2} < 0$$

indefinite => saddle points

Prob 6.

$$A = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} a-\lambda & 1 & 1 \\ 1 & a-\lambda & 1 \\ 1 & 1 & a-\lambda \end{vmatrix} = (a-\lambda)^3 + 1 + 1 - 3(a-\lambda)$$

$$= (a-\lambda)^3 - 3(a-\lambda) + 2$$

$t = \lambda - a$

$$t^3 - 3t + 2 = 0 \iff (t-1)^2(t+2) = 0$$

$$t = 1 \text{ or } t = -2.$$

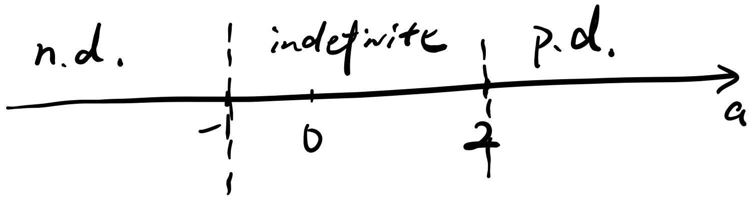
$$\iff \lambda = a+1 \text{ or } \lambda = a-2$$

$$\begin{array}{r} t^2 + t - 2 \\ t-1 \enclose{longdiv}{t^3 - 3t + 2} \\ \underline{t^3 - t^2} \\ t^2 - 3t + 2 \\ \underline{t^2 - t} \\ -2t + 2 \\ -2t + 2 \end{array}$$

A is p.d. $\iff$ all λ_i are positive

$$\iff \begin{array}{l} a+1>0 \\ a-2>0 \end{array}$$

$$\iff a>2$$



Prob 4 #2.

$$\vec{v}_1 = \begin{pmatrix} 0 \\ 2 \\ 1 \\ 0 \end{pmatrix} \quad \vec{v}_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \quad \vec{v}_3 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix}$$

① $\vec{u}_1 = \vec{v}_1$

② $$\vec{u}_2 = \vec{v}_2 - \frac{\vec{v}_2 \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} \cdot \vec{u}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} - \frac{(-2)}{5} \cdot \begin{pmatrix} 0 \\ 2 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -1/5 \\ 2/5 \\ 0 \end{pmatrix}$$

③ $$\vec{u}_3 = \vec{v}_3 - \frac{\vec{v}_3 \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} \vec{u}_1 - \frac{\vec{v}_3 \cdot \vec{u}_2}{\vec{u}_2 \cdot \vec{u}_2} \cdot \vec{u}_2$$

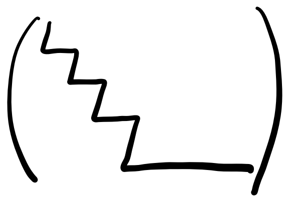
$$= \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix} - \frac{4}{5} \cdot \begin{pmatrix} 0 \\ 2 \\ 1 \\ 0 \end{pmatrix} - \overset{1/2}{\overset{\|}{\frac{3/5}{1+1/5}}} \cdot \begin{pmatrix} 1 \\ -1/5 \\ 2/5 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 \\ 2 - 8/5 + 1/10 \\ -4/5 - 1/5 \\ -1 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \\ -1 \\ -1 \end{pmatrix} \qquad \frac{1}{4} + \frac{1}{4} + 1 + 1 = \frac{5}{2}$$

Normalize $\vec{u}_1, \vec{u}_2, \vec{u}_3$, we get basis

$$\left\{ \frac{1}{\sqrt{5}} \cdot \vec{u}_1, \ \frac{1}{\sqrt{6/5}} \cdot \vec{u}_2, \ \frac{1}{\sqrt{5/2}} \cdot \vec{u}_3 \right\}.$$

$Col(A) = \mathbb{R}^m \iff$ column vectors span $\mathbb{R}^m$.

$\iff$ 

A. $\exists$ pivot in the last row.