Week 3. Thursday.

Last time: we show that. the solution set for $A\vec{x}=\vec{b}$ is $\vec{v}$ + solution set of $A\vec{x}=\vec{0}$ where $\vec{v}$ is a particular solution of $A\vec{x}=\vec{b}$.

Therefore if $A\vec{x}=\vec{b}$ has a unique solution, then $A\vec{x}=\vec{o}$ has a unique solution. (which must be $\vec{x}=\vec{0}$).

The converse is not necessarily true.

a.m. for $A\vec{x}=\vec{0}$ is

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$\Rightarrow$ $\vec{x}=0$ is the only solution. since there is no free variable.

a.m for $A\vec{x}=\vec{b}$ where $\vec{b}=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \boxed{1} \end{array}\right)$$

$\Rightarrow$ no solution since there is a pivot in last column.

In order for $A\vec{x}=\vec{b}$ to have a solution. we just need $\vec{b}=\begin{pmatrix}*\\*\\*\\*\\0\end{pmatrix}$.

Q: For this A. is it true that $A\vec{x}=\vec{b}$ has exactly one solution when $A\vec{x}=\vec{b}$ has a solution? Yes!

<u>Conclusion</u>: If $A\vec{x}=\vec{0}$ has a unique solution, then $A\vec{x}=\vec{b}$ has at most one solution.

Q: If $A\vec{x}=\vec{o}$ has ∞ solutions. how many solutions $A\vec{x}=\vec{b}$ has?

__Conclusion__: If $A\vec{x}=\vec{0}$ has ∞ solutions, then $A\vec{x}=\vec{b}$ either has no solutions, or has ∞ solutions.

simply use structure of solution set of nonhomogeneous system.

- Less trivial examples that are inconsistent:

a.m.

$$\left(\begin{array}{ccc|c} \boxed{1} & 0 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \boxed{1} \end{array}\right)$$

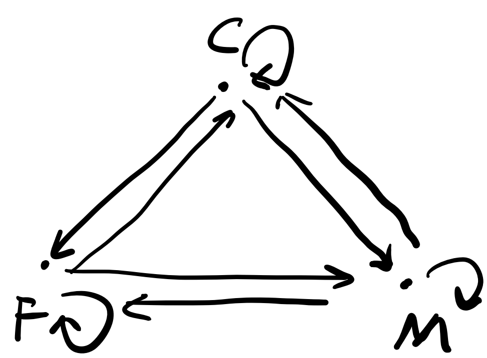
$A\vec{x}=\vec{0}$ ∞ solutions $\qquad$ $A\vec{x}=\vec{b}$ no solution.

1.6 Application.

Economic: input-output model says that each sector of economic pays the same amount of money with the amount of money they make:

Sector A B C.

Purchased by	Chemicals	Fuels	Machinery.
C	20%	80%	40%
F	30%	10%	40%
M.	50%	10%	20%



Q: What is the equilibrium pricing (total value) for each sector? P_C, P_F, P_M

For C. $P_C = 0.2P_C + 0.8P_F + 0.4P_M$ ①

$0.8P_C - 0.8P_F - 0.4P_M = 0$

F. $\quad P_F = 0.3P_C + 0.1P_F + 0.4P_M$. ②

M. $\quad P_M = 0.5P_C + 0.1P_F + 0.2P_M$. ③

a.m. $\left(\begin{array}{ccc|c} 8 & -8 & -4 & 0 \\ -3 & 9 & -4 & 0 \\ -5 & -1 & 8 & 0 \end{array}\right)$

③ ⇝ ③ + ① + ②

$\left(\begin{array}{ccc|c} 8 & -8 & -4 & 0 \\ -3 & 9 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

① Q: Is this purely lucky or coincidence?

② Can you ever find a situation where there are two zero rows in e.f.?

② ⇝ ② + ① $\cdot \frac{3}{8}$

$\left(\begin{array}{ccc|c} \boxed{8} & -8 & -4 & 0 \\ 0 & \boxed{9-3} & -4-\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

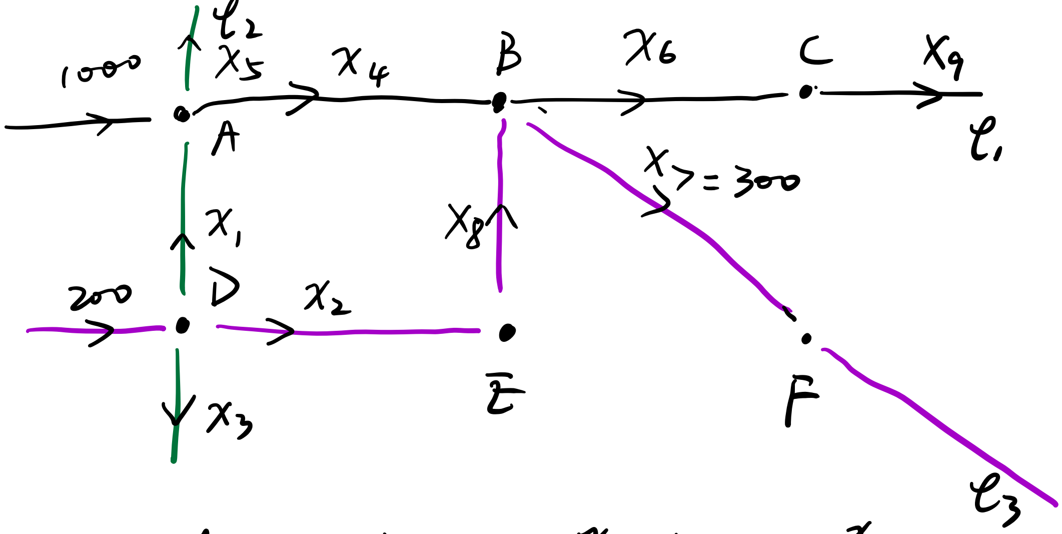
free variables.

$$\begin{cases} P_C = P_F + \frac{1}{2}P_M = \frac{17}{12}P_M \\ P_F = \frac{11}{12}P_M \\ P_M \text{ is free} \end{cases}$$

$\vec{P} = \begin{pmatrix} \frac{17}{12} \\ \frac{11}{12} \\ 1 \end{pmatrix} \cdot P_M$ is the parametric vector form.

Chemical Equations (Read for Yourself).

Network Flow: Subway system in a city.



For node A: $\quad 1000 = x_5 + x_4 - x_1$

node B: $\quad x_4 + x_8 = x_6 + 300$

C $\quad$:

D $\quad$:

E $\quad$: