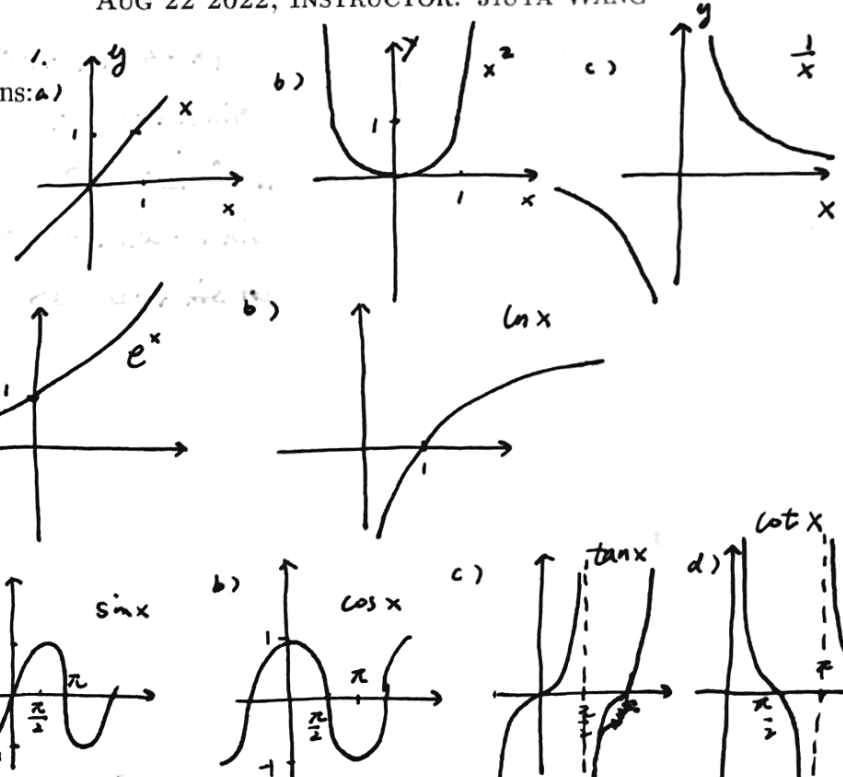## Problem 1 : Graph of Functions

Sketch the graph of the following functions:

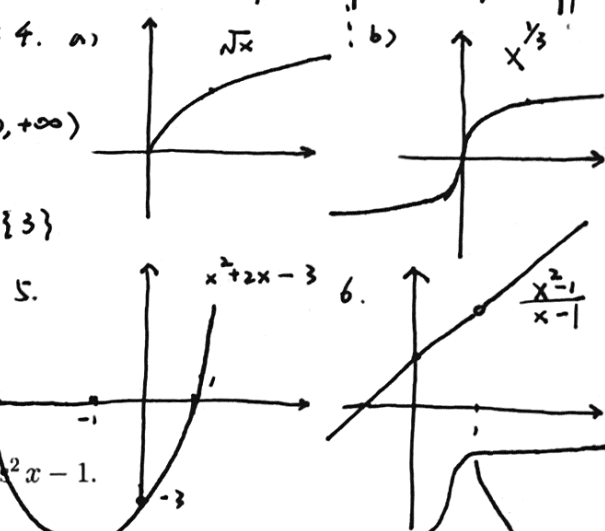1. x, x^2, $\frac{1}{x}$

2. e^x, $\ln x$

3. $\sin x$, $\cos x$, $\tan x$, $\cot x$

4. $\sqrt{x}$, $x^{1/3}$

5. $x^2 + 2x - 3$

6. $\frac{x^2-1}{x-1}$

7. $\frac{1}{2x-1}$

8. $\frac{3x+2}{x-1}$

9. $f(x) = \begin{cases} x+1, x \geq 0 \\ 1, x < 0. \end{cases}$

10. (*) $\sqrt{x^2-1}$

Problem 2: Domain and Range

Determine the domain and range for the following functions:

1. 1,2,3,4 in Problem 1

2. $x^2 + 2x - 3$, $x^2 - 4x + 4$ D: $\mathbb{R}$, $\mathbb{R}$; R: $[-4, +\infty)$, $[0, +\infty)$

3. $\ln(x^2 + 2x - 3)$ D: $(-\infty, -3) \cup (1, +\infty)$; R: $\mathbb{R}$

4. $\frac{1}{2x-1}$, $\frac{3x+2}{x-1}$ D: $\mathbb{R}\setminus\{\frac{1}{2}\}$, $\mathbb{R}\setminus\{1\}$; R: $\mathbb{R}\setminus\{0\}$, $\mathbb{R}\setminus\{3\}$

5. $\frac{1}{\sqrt{\sin x}}$ D: $\bigcup_{k\in\mathbb{Z}}(2k\pi, 2k\pi + \pi)$; R: $[1, +\infty)$

6. $\frac{x^2-1}{x-1}$ D: $\mathbb{R}\setminus\{1\}$; R: $\mathbb{R}\setminus\{2\}$

7. $\sin^2(x/\pi)$ D: $\mathbb{R}$; R: $[0,1]$

8. (*) $\cos(2x) + 2\sin x$ Hint: $\cos(2x) = 1 - 2\sin^2 x = 2\cos^2 x - 1$.
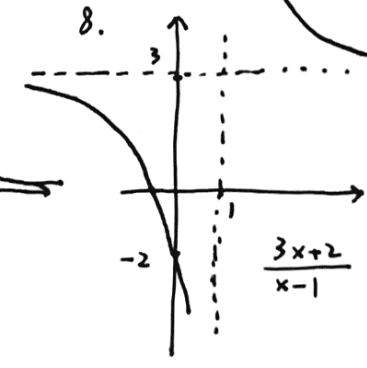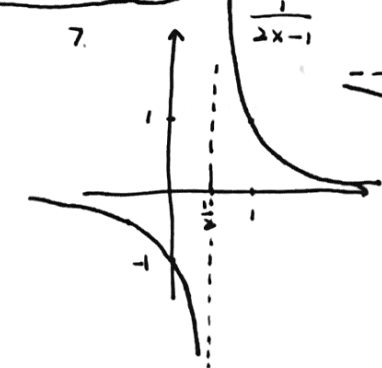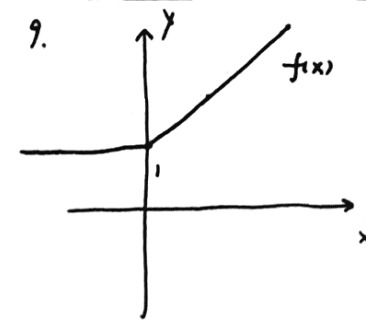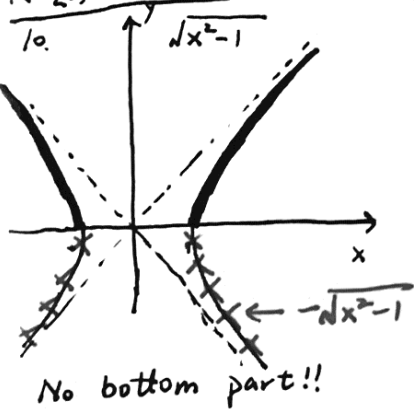   D: $\mathbb{R}$; R: $[-3, \frac{3}{2}]$

Handwritten solutions:

1. $x, x^2, \frac{1}{x}$
D: $\mathbb{R}, \mathbb{R}, \mathbb{R}\setminus\{0\}$
R: $\mathbb{R}, [0, +\infty), \mathbb{R}\setminus\{0\}$

2. e^x $\ln x$
D: $\mathbb{R}$ $(0, \infty)$
R: $(0, +\infty)$ $\mathbb{R}$

3. $\sin x$, $\cos x$
D: $\mathbb{R}$ $\mathbb{R}$
R: $[-1, 1]$ $[-1, 1]$
$\tan x$, $\cot x$
D: $\mathbb{R}\setminus\{k\pi \mid k\in\mathbb{Z}\}$,
R: $\mathbb{R}$, $+\frac{\pi}{2}$ $\mathbb{R}$

4. $\sqrt{x}$, $x^{1/3}$
D: $[0, +\infty)$, $\mathbb{R}$
R: $[0, +\infty)$, $\mathbb{R}$

1. a) b) x^2 c) $\frac{1}{x}$

2. a) e^x b) $\ln x$

3. a) $\sin x$ b) $\cos x$ c) $\tan x$ d) $\cot x$

4. a) $\sqrt{x}$ b) $x^{1/3}$

5. x^2+2x-3 6. $\frac{x^2-1}{x-1}$

10. $\sqrt{x^2-1}$ $\leftarrow -\sqrt{x^2-1}$ No bottom part!!

9. $f(x)$

7. $\frac{1}{2x-1}$

8. $\frac{3x+2}{x-1}$

Problem 3: Zeros of Functions Determine the x such $f(x) = 0$ for the following $f(x)$

1. $f(x) = x^2 - 4x + 3$ $(x-3)(x-1) = 0 \Rightarrow x = 3$ or $x = 1$

2. $f(x) = \sin(2x)$ $\sin 2x = 0 \Rightarrow 2x = k\pi, k \in \mathbb{Z} \Rightarrow x = \frac{k}{2}\pi, k \in \mathbb{Z}$

3. $f(x) = \sqrt{e^x - 1}$ $\sqrt{e^x - 1} = 0 \Rightarrow e^x - 1 = 0 \Rightarrow e^x = 1 \Rightarrow x = 0$

4. $f(x) = \sin x - \cos x$ $\sin x - \cos x = 0 \Rightarrow \tan x = 1 \Rightarrow x = k\pi + \frac{\pi}{4}, k \in \mathbb{Z}$

5. $f(x) = \ln(\sin x)$ $\ln \sin x = 0 \Rightarrow \sin x = 1 \Rightarrow x = 2k\pi + \frac{\pi}{2}, k \in \mathbb{Z}$

Problem 4: Even and Odd function Determine whether the following function is even or odd function?

1. $|x|, x^2, x^3, 1/x, \sqrt{x}$ E, E, O, O, N/A

2. $e^{x^2}, \sin x, \cos x, \sin(x^2)$ E, O, E, E

3. $\sqrt{x^2 - 1}, \sqrt{x^3 - 1}$ E, N/A