**Problem 1: Evaluating Limit**
Determine the limit of the following function $f(x)$ at the given point:

*Plug in* →

1. $x-1$, $x^2$, $e^x$, $\sin x$, $\cos x$ at $x=a$    $a-1,\ a^2,\ e^a,\ \sin a,\ \cos a$

2. $e^{x^2-x+2}$ at $x=1$    $e^2$

3. $\frac{x^2-3x+2}{x-1}$ at $x=1$    $-1$    Factor  $x^2-3x+2=(x-2)(x-1)$

4. $\frac{\sqrt{x^2+1}-1}{x}$ at $x=0$    $0$   Multiply $\sqrt{x^2+1}+1$ on both top and bottom

5. $\frac{\sin(2x)}{\sin x}$ at $x=0$ (Hint: $\sin(2x)=2\sin x\cos x$)    $2$

6. $\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}}$ at $x=0$    $0$   Multiply $x^2$ on top & bottom

7. $\ln(1+x^2)$ at $x=0$    $0$

8. $\frac{x^n-1}{x-1}$ at $x=1$ (Hint: $x^n-1=(x-1)(x^{n-1}+x^{n-2}+\cdots+1)$)    $n$

9. $\frac{\frac{1}{x-1}-1}{x-2}$ at $x=2$    $-1$   Multiply  $x-1$   $\cdots\cdots$

10. $\sin x\cos x$ at $x=0$    $0$

11. $\frac{x-4}{\sqrt{x}-2}$ at $x=4$    $4$   Multiply $\sqrt{x}+2$ $\cdots$ ; Or Factor $x-4=(\sqrt{x}+2)\cdot(\sqrt{x}-2)$

12. $\frac{x^2-1}{x^3-1}$ at $x=1$    $\frac{2}{3}$   Factor like Q8

**Problem 2: Evaluate Left/Right Limit**
Determine the limit of the following function at the given point:

*"Plug in" & Find Sign)*
*( & Sometimes)*

1. $\frac{x^3-2x-1}{x+1}$ at $x=1$ from the left    $-1$

2. $\frac{|x-1|}{x-2}$ at $x=2$ from the left    $-\infty$    near $x=2$   $\frac{|x-1|}{x-2}=\frac{x-1}{x-2}$

3. $\frac{\sqrt{x-1}}{x-1}$ at $x=1$ from the right    $+\infty$

4. $\ln(x^2+2x)$ at $x=0$ from the right    $-\infty$

5. $\frac{|(1+x)^2-1|}{x}$ at $x=0$ from the left    $-2$    near $x=0$ (on the left) $f(x)=\frac{-(x^2+2x)}{x}$

6. $e^{x-1/x}$ at $x=0$ from the right    $0$

7. $\cos(1/x)$ at $x=0$ from the left    $DNE$

8. $1/\ln(x)$ at $x=0$ from the right    $0$

**Problem 3: Infinite Limit** Determine the vertical asymptote for the following function and write down a infinite limit (right/left/positive/negative) as the reason:

1. $\frac{x+1}{x-1}$  $x = 1$   $\lim\limits_{x \to 1^+} f(x) = +\infty$   $\lim\limits_{x \to 1^-} f(x) = -\infty$

2. $\ln(\frac{x+1}{x-1})$  $x = -1$   $\lim\limits_{x \to -1^-} f(x) = -\infty$

3. $\frac{x-2}{x^2-4}$  $x = -2$   $\lim\limits_{x \to -2^+} f(x) = +\infty$   $\lim\limits_{x \to -2^-} f(x) = -\infty$

4. $e^{\frac{x-2}{x^2-4}}$  $x = -2$   $\lim\limits_{x \to -2^+} f(x) = +\infty$

5. $\sqrt{\frac{x}{x-1}}$  $x = 1$   $\lim\limits_{x \to 1^+} f(x) = +\infty$

6. $\ln(\sin x)$  $x = 2k\pi$

   $2k\pi + \pi$   $\lim\limits_{x \to 2k\pi^+} f(x) = -\infty$

   $\lim\limits_{x \to (2k\pi+\pi)^-} f(x) = -\infty$

**Problem 4: Pattern Recognition** Look at the graph of the following graph for $f(x)$ and determine the answer:

1. What is the limit of $f(x)$ at $x = -1$?

2. What is the limit of $f(x)$ at $x = 1$?

3. What is the limit of $f(x)$ at $-2$?

4. What is the limit of $f(x)$ at $3$?
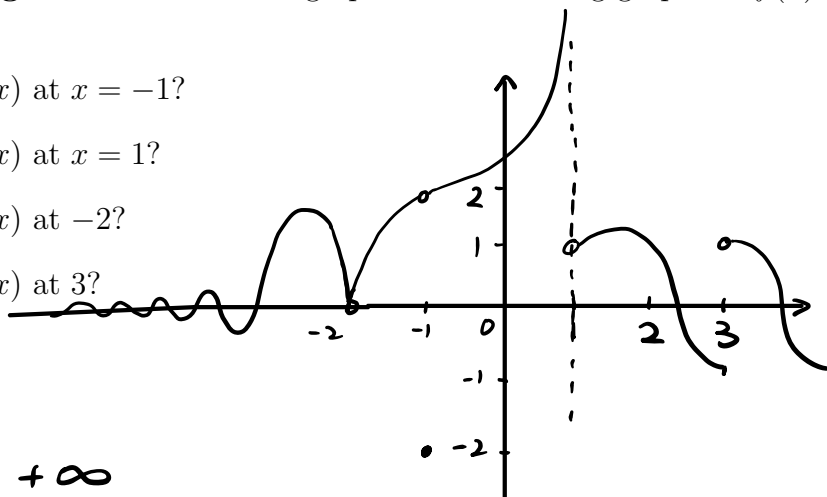


1. $\lim\limits_{x \to -1} f(x) = 2$

2. DNE   $\lim\limits_{x \to 1^-} f(x) = +\infty$

   $\lim\limits_{x \to 1^+} f(x) = 1$

3. $\lim\limits_{x \to -2} f(x) = 0$

4. DNE   $\lim\limits_{x \to 3^-} f(x) = -1$   $\lim\limits_{x \to 3^+} f(x) = 1$