Problem 1: Evaluating derivatives by definition

Determine if $f(x)$ is differentiable at the given point. If yes, give the derivative, if not, give the reason.

1. $x^2 + ax + b$ at $x = 2$ $4 + 2a$
2. $\frac{1}{x(x-1)}$ at $x = 2$ $-\frac{3}{4}$
3. $\sqrt{x+1}$ at $x = 0$ $\frac{1}{2}$
4. $\sin x$ at $x = 0$ 1
5. $\cos x$ at $x = 0$ 0
   (Hint: use the formula $1 - \cos x = 2\sin^2(x/2)$)
6. $\frac{1}{x^2}$ at $x = 1$ -2
7. $f(x) = \begin{cases} 0, x \leq 0 \\ x^2, x > 0 \end{cases}$ at $x = 0$ 0
8. $f(x) = \begin{cases} x^2 + 3x + 2, x < 1 \\ 5x - 1, x \geq 1 \end{cases}$ at $x = 1$ Not differentiable (not continuous)

Problem 2: Evaluating derivative function by definition

Determine the derivative function of the following function by definition:

1. $f(x) = x^n$ ($n > 0$ is a fixed integer) $n \cdot x^{n-1}$
2. $f(x) = \sqrt{x}$ $\frac{1}{2\sqrt{x}}$
3. $f(x) = \sin x$ $\cos x$
   (Hint: $\sin(a + b) = \sin a \cos b + \sin b \cos a$)
4. $f(x) = \cos x$ $-\sin x$
   (Hint: $\cos(a + b) = \cos a \cos b - \sin a \sin b$)

Will be run in class

Problem 3: Compute the derivatives Determine the derivatives from power/sum/subtract/product/qu rule

1. $x \sin x$ $\quad x \cdot \cos x + \sin x$
2. $\tan x$ $\quad \frac{1}{\cos^2 x}$
3. x^n (n is arbitrary integer) $\quad n \cdot x^{n-1}$
4. $\frac{x^2+1}{x-1}$ $\quad \frac{x^2-2x-1}{(x-1)^2}$
5. $\sin^2 x - \cos^2 x$ $\quad 4 \sin x \cdot \cos x$
6. $\frac{1}{x^2 - \tan x}$ $\quad \frac{-2x + \frac{1}{\cos^2 x}}{(x^2 - \tan x)^2}$

Problem 4: Compute the second-order derivatives Look at the graph of the following graph for $f(x)$ and determine the answer:

1. $f(x) = x^5 + 4x^3 - 2x + 1$ $\quad f'(x) = 5x^4 + 12x^2 - 2, \; f''(x) = 20x^3 + 24x$
2. $f(x) = x^2 \sin x$ $\quad f'(x) = 2x\sin x + x^2 \cos x, \; f''(x) = -x^2 \sin x + 4x\cos x + 2\sin$
3. $f(x) = \frac{1}{x-1}$ $\quad f'(x) = -\frac{1}{(x-1)^2} \quad f''(x) = \frac{2}{(x-1)^3}$
4. $f(x) = \frac{\sin x}{x+1}$

$$f'(x) = \frac{(x+1) \cdot \cos x - \sin x}{(x+1)^2} \qquad f''(x) = \frac{-(x+1)^3 \cdot \sin x - 2(x+1)^2 \cos x + 2(x+1) \sin}{(x+1)^4}$$