Problem 1: Derivatives Determine the derivatives and second order derivatives of the following functions:

1. e^{x^2-2x}
2. $x^{\ln x}$
3. $(\ln x)^x$
4. $(\sin(e^x))^3$
5. $\frac{e^x-e^{-x}}{\sqrt{e^x}}$
6. $e^{e^x}+e^{x^e}$
7. $\frac{1+\ln(x^3)}{x}$
8. $5^{\cos x+\sin x}$
9. $\log_5(e^x)$
10. $\log_5(1+x^{100})$
11. $\arcsin(e^x)$
12. $\frac{\arctan(x^2)-x^2}{\ln x}$
13. $\arccos(x^2-2x)\cdot e^x$
14. $(\frac{x}{e})^x\cdot\sqrt{2\pi x}$

Problem 2: Implicit Differentiation
Determine the derivatives $\frac{dy}{dx}$ of the functions $y=y(x)$.

1. $y+x=e^{xy}$
2. $\frac{y}{x}+\frac{x}{y}=\ln(x^2)$
3. $x^2y+xy^2-e^y=0$
4. $\frac{\sin x}{y}+y^2=3^x$

Prob 1.

1. $f' = e^{(x^2-2x)} \cdot (2x-2)$

$f'' = e^{(x^2-2x)} \cdot (2x-2)^2 + e^{(x^2-2x)} \cdot 2$

2. $f' = x^{\ln x} \cdot 2 \cdot \frac{\ln x}{x}$

$f'' = x^{\ln x} \cdot \left(2\frac{\ln x}{x}\right)^2 + x^{\ln x} \cdot 2 \cdot \frac{1-\ln x}{x^2}$

3. $f' = ((\ln x))^x \cdot \left(\ln\ln x + \frac{1}{\ln x}\right)$

$f'' = (\ln x)^x \cdot \left(\ln\ln x + \frac{1}{\ln x}\right)^2 + (\ln x)^x \cdot \left[\frac{1}{x \cdot \ln x} - \frac{1}{(\ln x)^2 \cdot x}\right]$

4. $f' = 3 \cdot [\sin(e^x)]^2 \cdot \cos(e^x) \cdot e^x$

$f'' = 3 \cdot 2 \cdot \sin(e^x) \cdot [\cos(e^x)]^2 \cdot e^{2x}$

$+ 3 \cdot [\sin(e^x)]^2 \cdot [-\sin(e^x)] \cdot e^{2x} + 3 \cdot [\sin(e^x)]^2 \cdot \cos(e^x) \cdot e^{2x}$

5. $f' = \frac{1}{2} e^{\frac{x}{2}} + \frac{3}{2} e^{-\frac{3}{2}x}$ $\quad f'' = \frac{1}{4} e^{\frac{x}{2}} - \frac{9}{4} e^{-\frac{3}{2}x}$

6. $f' = e^{(e^x)} \cdot e^x + e^{(x^e)} \cdot e \cdot x^{e-1}$

$f'' = e^{(e^x)} \cdot e^{2x} + e^{(e^x)} \cdot e^x + e^{(x^e)} \cdot e^2 \cdot x^{2(e-1)} +$

$e^{(x^e)} \cdot e \cdot (e-1) \cdot x^{e-2}$

7. $f' = \frac{2 - 3\ln x}{x^2}$ $\quad f'' = \frac{-3x - x^2 \cdot (2 - 3\ln x)}{x^4}$

8. $f' = 5^{\cos x + \sin x} \cdot \ln 5 \cdot (-\sin x + \cos x)$

$f'' = 5^{\cos x + \sin x} \cdot (\ln 5)^2 \cdot (-\sin x + \cos x)^2 +$

$5^{\cos x + \sin x} \cdot \ln 5 \cdot (-\cos x - \sin x)$

9. $f' = \frac{1}{\ln 5}$ $\qquad f'' = 0$

10. $f' = \frac{100\cdot x^{99}}{(1+x^{100})\ln 5}$ $\qquad f'' = \frac{100}{\ln 5}\cdot\frac{99x^{98}\cdot(1+x^{100}) - x^{99}\cdot 100\cdot x^{99}}{(1+x^{100})^2}$

$$= \frac{100}{\ln 5}\cdot\frac{99x^{98} - x^{198}}{(1+x^{100})^2}$$

11. $f' = \frac{e^x}{\sqrt{1-e^{2x}}}$ $\qquad f'' = \frac{e^x\cdot\sqrt{1-e^{2x}} - e^x\cdot\frac{1}{2}\cdot(1-e^{2x})^{-\frac{1}{2}}\cdot(-2e^{2x})}{(1-e^{2x})}$

12. $f' = \frac{\left[\frac{1}{1+x^4}\cdot 2x - 2x\right]\cdot\ln x - (\arctan(x^2) - x^2)\cdot\frac{1}{x}}{(\ln x)^2}$

$$= \frac{-2\cdot x^5}{(1+x^4)\ln x} - \frac{\arctan(x^2) - x^2}{x\cdot(\ln x)^2}$$

$$f'' = \frac{-10x^4\cdot(1+x^4)\cdot\ln x + 2x^5\cdot\left[4x^3\cdot\ln x + \frac{1+x^4}{x}\right]}{(1+x^4)^2(\ln x)^2}$$

$$- \frac{\left[\frac{2x}{1+x^4} - 2x\right]\cdot x\cdot(\ln x)^2 - \left[\arctan(x^2) - x^2\right]\cdot\left[(\ln x)^2 + 2\cdot\ln x\right]}{x^2\cdot(\ln x)^4}$$

13. $f' = \frac{-1}{\sqrt{1-(x^2-2x)^2}}\cdot(2x-2)\cdot e^x + \arccos(x^2-2x)\cdot e^x$

$$f'' = -2\cdot\frac{e^x\cdot x\cdot\sqrt{1-(x^2-2x)^2} - (x-1)\cdot e^x\cdot\frac{1}{2}\cdot\left[1-(x^2-2x)^2\right]^{-\frac{1}{2}}\cdot\left[-2(x^2-2x)\right]\cdot(2x-2)}{1-(x^2-2x)^2}$$

$$+ \frac{-1}{\sqrt{1-(x^2-2x)^2}}\cdot(2x-2)\cdot e^x + \arccos(x^2-2x)\cdot e^x$$

$e^{x\cdot\ln\left(\frac{x}{e}\right)}$

14. $f' = \left(\frac{x}{e}\right)^x\cdot\sqrt{2\pi}\cdot\frac{1}{2}\cdot x^{-\frac{1}{2}} + \sqrt{2\pi x}\cdot\left(\frac{x}{e}\right)^x\cdot\ln x$

$$= \left(\frac{x}{e}\right)^x\cdot\sqrt{2\pi x}\cdot\left[\frac{1}{2x} + \ln x\right]$$

$$f'' = \left(\frac{x}{e}\right)^x \cdot 2\pi x \cdot \left[\frac{1}{2x} + \ln x\right]^2 \cdot$$

$$+ \left(\frac{x}{e}\right)^x \cdot \sqrt{2\pi} \cdot \frac{1}{2} x^{-\frac{1}{2}} \cdot \left[\frac{1}{2x} + \ln x\right]$$

$$+ \left(\frac{x}{e}\right)^x \cdot \sqrt{2\pi x} \cdot \left[-\frac{1}{2x^2} + \frac{1}{x}\right]$$

Prob 2. 1) $y' + 1 = e^{xy} \cdot (y + x \cdot y')$

$$\Rightarrow \quad y' \cdot [1 - x \cdot e^{xy}] = y \cdot e^{xy} - 1$$

$$y' = \frac{y \cdot e^{xy} - 1}{1 - x \cdot e^{xy}}$$

2) $$\frac{y' \cdot x - y \cdot 1}{x^2} + \frac{1 \cdot y - x \cdot y'}{y^2} = \frac{2}{x}$$

$$\Rightarrow \quad y' \cdot \left[\frac{1}{x} - \frac{x}{y^2}\right] = \frac{2}{x} - \frac{1}{y} + \frac{y}{x^2}$$

$$y' = \frac{\frac{2}{x} - \frac{1}{y} + \frac{y}{x^2}}{\frac{1}{x} - \frac{x}{y^2}} = \frac{2 \cdot xy^2 - x^2 y + y^2}{xy^2 - x^3}$$

3) $2x \cdot y + x^2 \cdot y' + y^2 + x \cdot 2y \cdot y' - e^y \cdot y' = 0$

$$\Rightarrow \quad y' [x^2 + 2xy - e^y] = -y^2 - 2xy$$

$$y' = -\frac{y^2 + 2xy}{x^2 + 2xy - e^y}$$

4) $\frac{(\cos x)\cdot y - (\sin x)\cdot y'}{y^2} + 2y \cdot y' = 3^x \cdot \ln 3$

$$\Rightarrow y' \cdot \left[2y - \frac{\sin x}{y^2} \right] = 3^x \cdot \ln 3 - \frac{\cos x}{y}$$

$$y' = \frac{3^x \cdot \ln 3 - \frac{\cos x}{y}}{2y - \frac{\sin x}{y^2}}$$