Problem 1 : Simplification

Simplify or compute the following expressions:

1. $\begin{pmatrix} 3t \\ 4 \end{pmatrix} + 2\begin{pmatrix} t+1 \\ t-1 \end{pmatrix}$ $\begin{pmatrix} 5t+2 \\ 2t+2 \end{pmatrix}$
2. $\begin{pmatrix} 1 \\ \cos\theta \\ \sin\theta \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \cos\theta \\ \sin\theta \end{pmatrix}$ 2
3. $\begin{pmatrix} 1 \\ \cos\theta \\ \sin\theta \end{pmatrix} \times \begin{pmatrix} t \\ t\cos\theta \\ t\sin\theta \end{pmatrix}$ $\vec{0}$
4. $\left\| \begin{pmatrix} r\sin\theta\sin\omega \\ r\sin\theta\cos\omega \\ r\cos\theta \end{pmatrix} \right\|$ $|r|$

Problem 2

Page 16, Problem 2: Determine whether the expressions are legal or not, and if legal, determine the expression is a vector or a number:

1. $(\vec{a}\cdot\vec{b})\times\vec{c}$; ✗
2. $(\vec{a}\times\vec{b})\cdot\vec{c}$; ✓
3. $\left\|\vec{a}\times\vec{b}\right\|$ ✓

Problem 3: more simplification Page 17 Problem 13, 15

13. a) $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = \|\vec{a}\|^2 - \|\vec{b}\|^2$

b) $\vec{a}\perp\vec{b}$: $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b} = \|\vec{a}\|^2 + \|\vec{b}\|^2$

since $\vec{a}\cdot\vec{b} = 0$

c) $\vec{a}\perp\vec{b}$: $\|\vec{a}-\vec{b}\|^2 = \|\vec{a}\|^2 + \|\vec{b}\|^2$

15. a) $(\vec{a}+\vec{b})\times(\vec{a}+\vec{b}) = \vec{a}\times\vec{a} + \vec{a}\times\vec{b} + \vec{b}\times\vec{a} + \vec{b}\times\vec{b} = \vec{0} + \vec{a}\times\vec{b} - \vec{a}\times\vec{b} + \vec{0} = 0$

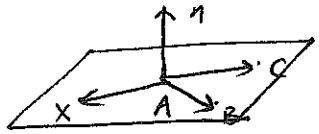
b) $\vec{0}$ c) $2\vec{a}\times\vec{b}$ d) $2\vec{a}\times\vec{c} - 2\vec{a}\times\vec{b}$ e) $\|\vec{a}\|^2 - \|\vec{b}\|^2 - \|\vec{c}\|^2 + 2\vec{b}\cdot\vec{c}$

Problem 4: Defining equation for plane

1. l is the plane passing through A: $(1,2,0)$, B: $(0,0,1)$, C: $(0,1,0)$. What is the normal vector $\vec{n}$ of l?
2. Given arbitrary point X: (x,y,z) on the plane, what is $\vec{AX}\cdot\vec{n}$?
3. What is the defining equation of l?

1. $\vec{AB} = \begin{pmatrix}-1\\-2\\1\end{pmatrix}$ $\vec{AC} = \begin{pmatrix}-1\\-1\\0\end{pmatrix}$

$$\vec{n} = \vec{AB}\times\vec{AC} = \begin{vmatrix} i & j & k \\ -1 & -2 & 1 \\ -1 & -1 & 0 \end{vmatrix} = \begin{pmatrix}1\\-1\\-1\end{pmatrix}$$

2. 

$\vec{AX}\cdot\vec{n} = 0$

3. $\begin{pmatrix}x-1\\y-2\\z-0\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\-1\end{pmatrix} = (x-1)-(y-2)-z = x-y-z+1 = 0$