Math 234 Worksheet ~~4~~ 3 ~~27 Sept 2016~~ 6 Feb 2017, TA: Jiuya Wang

Problem 1 : Classification of Quadratic Form

Firstly use completing the square to classify the following quadratic forms;
then use $4AC - B^2$ method to do it again;
finally determine the zero sets for each of them:

- The first 5 is in good form so no need to complete the square
- $4AC - B^2 < 0$ indefinite
- $4AC - B^2 = 0$ semi-definite
- $4AC - B^2 > 0$ $\begin{cases} A > 0 & \text{positive definite} \\ A < 0 & \text{negative definite} \end{cases}$

zero sets on last page:

1. $f(x, y) = x^2 + 2y^2$ positive definite
2. $f(x, y) = x^2 - y^2$ indefinite
3. $f(x, y) = -x^2 - y^2$ negative definite
4. $f(x, y) = xy$ indefinite
5. $f(x, y) = x^2$ semi-definite
6. $f(x, y) = x^2 - 4xy + 3y^2 = x^2 + 2\cdot x\cdot(-2y) + (-2y)^2 - (-2y)^2 + 3y^2 = [x + (-2y)]^2 - y^2$ indefinite
7. $f(x, y) = 9x^2 - 36xy + 81y^2 = 9[x^2 - 4xy + 9y^2]$
   $= 9[x^2 + 2\cdot x\cdot(-2y) + (-2y)^2 - (-2y)^2 + 9y^2]$
   $= 9[(x-2y)^2 + 5y^2] = 9(x-2y)^2 + 45y^2$ positive definite
8. $f(x, y) = xy + y^2$
9. $f(x, y) = x^2 + 2xy$
10. $f(x, y) = \frac{1}{2}x^2 - xy + y^2$

8. $xy + y^2$
$= y^2 + 2\cdot y\cdot \frac{x}{2} + (\frac{x}{2})^2 - (\frac{x}{2})^2$
$= (y + \frac{x}{2})^2 - (\frac{x}{2})^2$ indefinite

9. $x^2 + 2xy$
$= x^2 + 2\cdot x\cdot y + y^2 - y^2$
$= (x+y)^2 - y^2$
indefinite

10. $\frac{1}{2}x^2 - xy + y^2$
$= \frac{1}{2}[x^2 - 2xy + 2y^2]$
$= \frac{1}{2}[x^2 + 2\cdot x\cdot(-y) + (-y)^2 - (-y)^2 + 2y^2]$
$= \frac{1}{2}[(x-y)^2 + y^2]$ positive definite
$= \frac{1}{2}(x-y)^2 + \frac{1}{2}y^2$

Problem 2: Domain

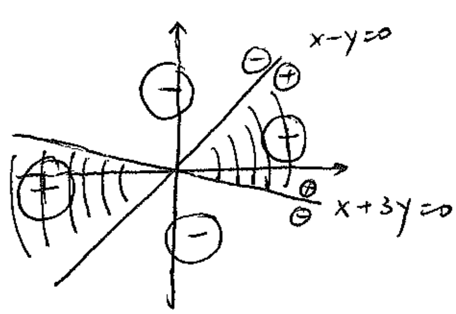
Find the largest domain where the functions can be defined:

1. $f(x, y) = \sqrt{9 - x^2} + \sqrt{y^2 - 4}$
2. $f(x, y) = \frac{1}{\sqrt{16 - x^2 - 4y^2}}$
3. $\sqrt{x^2 + 2xy - 3y^2}$

1. $\begin{cases} 9 - x^2 \geq 0 \\ y^2 - 4 \geq 0 \end{cases} \Rightarrow \begin{cases} |x| \leq 3 \\ |y| \geq 2 \end{cases}$

2. $16 - x^2 - 4y^2 > 0$

3. $(x^2 + 2xy - 3y^2) = (x+3y)(x-y) \geq 0$



$\begin{cases} x + 3y \geq 0 \\ x - y \geq 0 \end{cases}$ or $\begin{cases} x + 3y \leq 0 \\ x - y \leq 0 \end{cases}$

Problem 1:

zero set:

1. $x^2+2y^2=0 \Rightarrow x=0 \quad y=0$

2. $x^2-y^2=0=(x+y)(x-y) \Rightarrow \quad x+y=0$ or $x-y=0$

3. $-x^2-y^2=0 \Rightarrow x=0 \quad y=0$

4. $xy=0 \Rightarrow \quad x=0$ or $y=0$

5. $x^2=0 \Rightarrow x=0$

6. $[x+(-2y)]^2-y^2=(x-3y)(x-y) \Rightarrow \quad x-3y=0$ or $x-y=0$

7. $9(x-2y)^2+45y^2=0 \Rightarrow \quad x-2y=0$ and $y=0 \Rightarrow x=0 \; y=0$

8. $(y+\frac{x}{2})^2-(\frac{x}{2})^2=y\cdot(x+y) \Rightarrow y=0$ or $x+y=0$

9. $x^2+2xy=x\cdot(x+2y)=0 \Rightarrow \quad x=0$ or $x+2y=0$

10. $\frac{1}{2}(x-y)^2+\frac{1}{2}y^2=0 \Rightarrow \quad x-y=0$ and $y=0 \Rightarrow x=0 \; y=0$

Summary:

Type	General Form	$4AC-B^2$	Zero Set	Function Value
Indefinite	$+(\quad)^2-(\quad)^2$	$-$	2 lines	Positive and Negative
Semi-definite	$(\quad)^2$ or $-(\quad)^2$	0	1 line	Positive or Negative
Positive-definite	$+(\quad)^2+(\quad)^2$	$+$ $(A>0)$	$(0,0)$	Positive
Negative-definite	$-(\quad)^2-(\quad)^2$	$+$ $(A<0)$	$(0,0)$	Negative

※: To tell between Positive-definite and Negative-definite, you look at sign of A