Problem 1: Gradient

Compute the gradient of the following functions then determine that at the given point, in which direction the function **increases** the fastest, and in which direction the function **decreases** the fastest, and in which direction the function **remains** the same ?

1. $f(x,y) = x^2 + 3xy^2$; $x = 1, y = 1$
2. $f(x,y) = 100 - x^2 - 3y^3$; $x = 2, y = 1$

1. $\nabla f = \begin{pmatrix} 2x + 3y^2 \\ 6xy \end{pmatrix}$ $\nabla f\big|_{(1,1)} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$

so in direction $\begin{pmatrix} 5 \\ 6 \end{pmatrix}$ f increases the fastest, in direction $\begin{pmatrix} -5 \\ -6 \end{pmatrix}$, f decreases the fastest, in direction $\begin{pmatrix} 6 \\ -5 \end{pmatrix}$ or $\begin{pmatrix} -6 \\ 5 \end{pmatrix}$ f remains the same.

2. $\nabla f = \begin{pmatrix} -2x \\ -9 \cdot y^2 \end{pmatrix}$ $\nabla f\big|_{(2,1)} = \begin{pmatrix} -4 \\ -9 \end{pmatrix}$

so in direction $\begin{pmatrix} -4 \\ -9 \end{pmatrix}$, ... increases ...

in direction $\begin{pmatrix} 4 \\ 9 \end{pmatrix}$, ... decreases ..... in direction $\begin{pmatrix} -9 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} 9 \\ -4 \end{pmatrix}$, ... remains ...

Problem 2: Tangent Plane Revisited

We learned that: given $f(x,y,z)$, the level set is a surface, and the gradient is perpendicular to the tangent plane of the level set. Please use this idea to compute the tangent plane of the following surface:

1. $x^2 + 3y^2 + z^3 = 5$; $x = 1, y = 1, z = 1$
2. $x^2 + y^2 + z^2 = 1$; $x = 1, y = 0, z = 0$

1. Consider the surface as level set at 0 of

$f(x,y,z) = x^2 + 3y^2 + z^3 - 5$

then $\nabla f = \begin{pmatrix} 2x \\ 6y \\ 3z^2 \end{pmatrix}$ $\nabla f\big|_{(1,1,1)} = \begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$ is the normal vector of tangent plane.

so that. the plane is $\begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-1 \\ z-1 \end{pmatrix} = 2(x-1) + 6\cdot(y-1) + 3(z-1) = 0$

$\Leftrightarrow 2x + 6y + 3z = 11$

2. Consider the surface as level set at 0 of

$f(x,y,z) = x^2 + y^2 + z^2 - 1$

$\nabla f = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$ $\nabla f\big|_{(1,0,0)} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ is the normal vector of tangent plane.

so the plane is $\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-0 \\ z-0 \end{pmatrix} = 2\cdot(x-1) + 0 + 0 = 0$

$\Leftrightarrow x = 1$