MATH 234 WORKSHEET 7 MAR 6 2017, TA: JIUYA WANG

Problem 1 : Higher Partial Derivatives
For following functions:
compute higher order partial derivatives;

1. $f(x, y) = x^3y^2 + y^5$;
2. $f(x, y) = x \sin y$
3. $x^2 + 4y^2 + 16z^2 - 64 = 0$ where $z = z(x, y)$ is an implicit function of x and y.

1. $f_x = 3x^2y^2$ $f_{xx} = 6xy^2$ $f_{xy} = f_{yx} = 6x^2y$

$f_y = 2yx^3 + 5y^4$ $f_{yy} = 2x^3 + 20y^3$

2. $f_x = \sin y$ $f_{xx} = 0$ $f_{xy} = f_{yx} = \cos y$

$f_y = x \cdot \cos y$ $f_{yy} = x \cdot (-\sin y)$

3. $F(x,y,z) = x^2 + 4y^2 + 16z^2 - 64$

$F_x = 2x$
$F_y = 8y$
$F_z = 32z$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{x}{16z}$$

$$z_{xx} = -\frac{1}{16} \cdot \left(\frac{x}{z}\right)_x = -\frac{1}{16} \cdot \frac{z - x \cdot \frac{\partial z}{\partial x}}{z^2}$$

$$= -\frac{1}{16} \cdot \frac{z - x \cdot (-\frac{x}{16z})}{z^2}$$

$$= -\frac{16z^2 + x^2}{16^2 z^3}$$

$$z_{xy} = z_{yx} = -\frac{x}{16} \cdot \left(\frac{1}{z}\right)_y$$

$$= -\frac{x}{16} \cdot -\frac{1}{z^2} \cdot \frac{\partial z}{\partial y}$$

$$= \frac{x}{16z^2} \cdot -\frac{y}{4z} = -\frac{xy}{64z^3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{y}{4z}$$

$$z_{yy} = -\frac{1}{4} \cdot \left(\frac{y}{z}\right)_y = -\frac{1}{4} \cdot \frac{z - y \cdot \frac{\partial z}{\partial y}}{z^2}$$

$$= -\frac{1}{4} \frac{z - y(-\frac{y}{4z})}{z^2} = \frac{-(4z^2 + y^2)}{4^2 z^3}$$

Problem 2: Finding a function from its derivatives
1. Determine if the vector of functions $\vec{F}$ is $\vec{\nabla} f = \vec{F}$ for some f;
2. Determine f if such an f exists.

1. $\vec{F} = \begin{pmatrix} 6xy + 4e^y \\ 3x^2 + 4xe^y \end{pmatrix}$
2. $\vec{F} = \begin{pmatrix} x^2 - 2xy^3 \\ (xy)^2 \end{pmatrix}$
3. $\vec{F} = \begin{pmatrix} ye^{xy} + y \cos x \\ xe^{xy} + \sin x \end{pmatrix}$

All functions are defined on whole plane!

1. $P_y = 6y + 4e^y = Q_x$

$f(x,y) = \int P \cdot dx = 3x^2y + 4xe^y + C(y)$

$f_y = 3x^2 + 4xe^y + C'(y) = Q(x,y)$

$\Rightarrow C'(y) = 0 \Rightarrow C(y) = C$

$f(x,y) = 3x^2y + 4xe^y + C$

2. $P_y = -6xy^2 \neq Q_x = 2xy^2$

No such f.

3. $P_y = e^{xy} + y \cdot e^{xy} \cdot x + \cos x$

$Q_x = e^{xy} + x \cdot e^{xy} \cdot y + \cos x$

$f(x,y) = \int P \cdot dx = e^{xy} + y \sin x + C(y)$

$f_y = x \cdot e^{xy} + \sin x + C'(y) = Q$

$\Rightarrow C'(y) = 0 \Rightarrow C(y) = C$

$f(x,y) = e^{xy} + y \cdot \sin x + C$