Problem 1 : Describe Region

For all the region:

1. Sketch the region D;
2. Write the iterated integral on this region.

1. The region bounded by $z = x^2 + y^2$ and $z = 4$;
2. The region in the first octant bounded by $x + y + z = 9$, $2x + 3y = 18$ and $x + 3y = 9$.
3. The region bounded by $x^2 + y^2 = 1$ and $z = 0$, $z = 5$.
4. The region in the first octant bounded by $x^2 + y^2 = a^2$, and $z = x + y$.
5. The region in the first octant bounded by $x^2 + y^2 + z^2 = 1$.
6. The region bounded by $x^2 + y^2 + z^2 = 2$ and $z = \sqrt{x^2 + y^2}$.

Problem 2 : Cylinder Coordinate

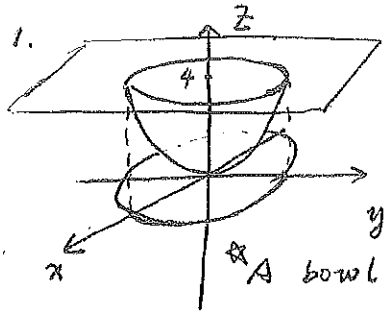
For the following region D, write the integral with cylinder coordinate.

1. Problem 1.1
2. Problem 1.3
3. Problem 1.4
4. Problem 1.6

Problem 3 : Sphere Coordinate
For the following region D, write the integral with sphere coordinate.

1. Problem 1.5
2. Problem 1.6

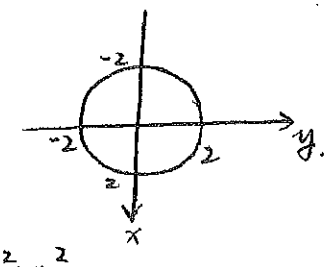
1.



A bowl

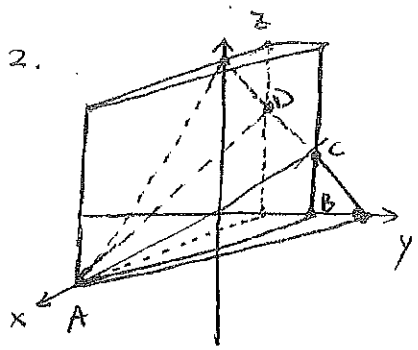
$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} f \, dz\, dy\, dx$$

Projection to xy-plane:



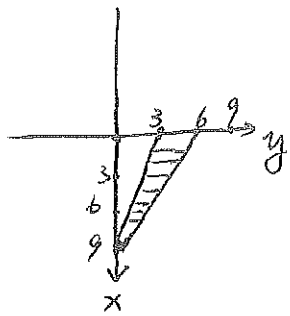
$$\begin{cases} z = x^2+y^2 \\ z = 4 \end{cases} \Rightarrow x^2+y^2 = 4$$

2.



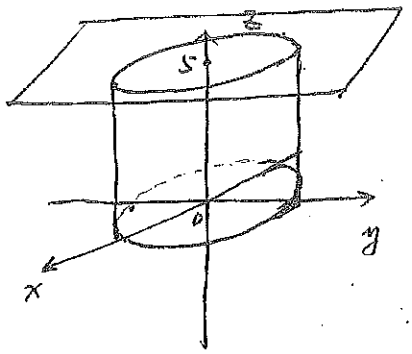
The shape is ABCD. like a wedge.

Projection to xy-plane:



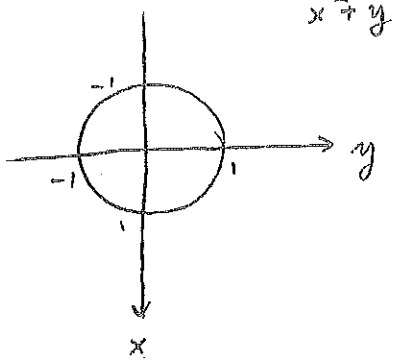
$$\int_0^9 \int_{\frac{9-x}{3}}^{\frac{18-2x}{3}} \int_0^{9-x-y} f \, dz\, dy\, dx$$

3.



A cylinder

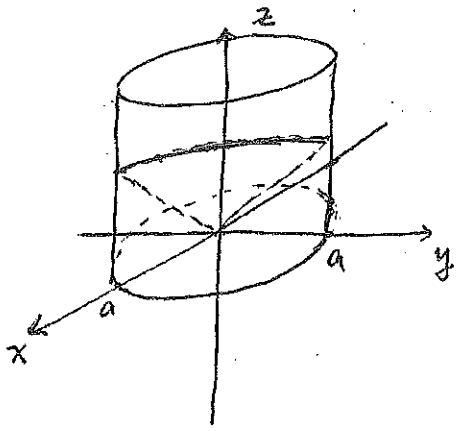
Projection:



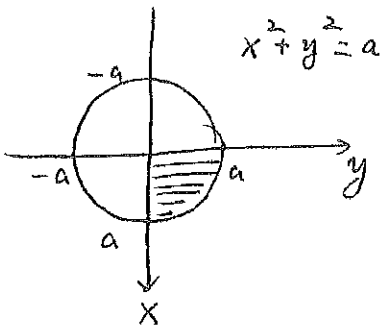
$x^2+y^2=1$

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^5 f \, dz\, dy\, dx$$

4.



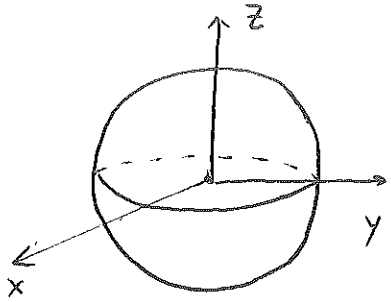
Projection:



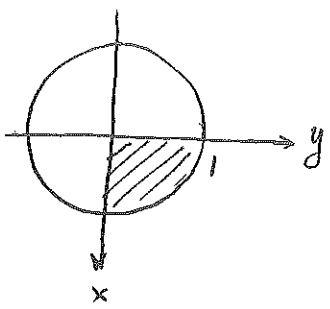
$x^2+y^2=a^2$

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{x+y} f \, dz\, dy\, dx$$

5.

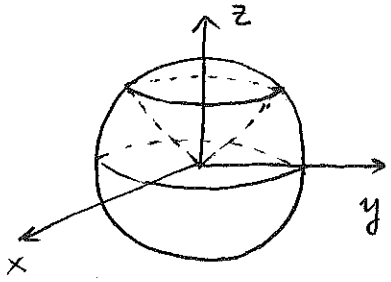


Projection



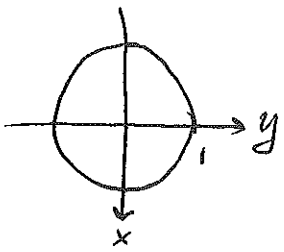
$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} f \cdot dz\,dy\,dx$$

6.



Projection

$$\begin{cases} x^2+y^2+z^2=2 \\ z=\sqrt{x^2+y^2} \end{cases} \Rightarrow x^2+y^2=1$$



$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} f\,dz\,dy\,dx$$

Cylinder:

1. $$\int_0^{2\pi} \int_0^2 \int_{r^2}^4 f \cdot r\,dz\,dr\,d\theta$$

2. $$\int_0^{2\pi} \int_0^1 \int_0^5 f \cdot r\,dz\,dr\,d\theta$$

3. $$\int_0^{\frac{\pi}{2}} \int_0^a \int_0^{r\cos\theta+r\sin\theta} f \cdot r \cdot dz\,dr\,d\theta$$

4. $$\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} f \cdot r\,dz\,dr\,d\theta$$

Sphere : 1. $\int_0^1 \int_0^{2\pi} \int_0^{\pi} f \cdot \rho^2 \sin\phi \; d\phi \; d\theta \; d\rho$

2. $\int_0^{\sqrt{2}} \int_0^{2\pi} \int_0^{\frac{\pi}{4}} f \cdot \rho^2 \sin\phi \; d\phi \; d\theta \; d\rho$