Problem 1 : Line integral of Functions
Compute the following line integral:

1. $f(x,y) = xy$, $C : \vec{\gamma}(t) = \begin{pmatrix} t \\ t^2 \end{pmatrix}, 0 \le t \le 1$
2. $f(x,y,z) = x^2 + y^2$, $C : \vec{\gamma}(t) = \begin{pmatrix} 3\cos t \\ 3\sin t \\ t \end{pmatrix}, 0 \le t \le 2\pi$
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1. $\vec{\gamma}'(t) = \begin{pmatrix} 1 \\ 2t \end{pmatrix}$ $\|\vec{\gamma}'(t)\| = \sqrt{1+4t^2}$

$\int_0^1 (t \cdot t^2) \cdot \sqrt{1+4t^2} \cdot dt$ $\quad \frac{u = 1+4t^2}{du = 8t\,dt}$

$\int_1^5 \frac{u-1}{4} \sqrt{u} \cdot \frac{du}{8} = \frac{1}{32}\int_1^5 u^{3/2} \cdot du - \frac{1}{32}\int_1^5 u^{1/2}\, du = \frac{1}{16} \cdot \left(\frac{5^{5/2}-1}{5} - \frac{5^{3/2}-1}{3} \right)$

2. $\vec{\gamma}'(t) = \begin{pmatrix} -3\sin t \\ 3\cos t \\ 1 \end{pmatrix}$ $\quad \|\vec{\gamma}'(t)\| = \sqrt{10}$

$\int_0^{2\pi} 3^2 \cdot \sqrt{10}\, dt = 9 \cdot \sqrt{10} \cdot 2\pi$

3. a) $\vec{\gamma}(\theta) = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$ $\quad 0 \le \theta \le \frac{\pi}{2}$

Arc Length: $\int_0^{\frac{\pi}{2}} \|\vec{\gamma}'(\theta)\| d\theta = \frac{\pi}{2}$

distance: $\sqrt{x^2+y^2}$ $\quad \vec{\gamma}'(\theta) = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$

Averge Distance: $\frac{\pi/2}{\pi/2} = 1$

$\int_0^{\frac{\pi}{2}} \sqrt{\cos^2\theta + \sin^2\theta} \cdot 1 \cdot d\theta = \frac{\pi}{2}$

b). $\int_0^{\frac{\pi}{2}} \theta \cdot \|\vec{\gamma}'(\theta)\| \cdot d\theta = \frac{\theta^2}{2}\Big|_0^{\frac{\pi}{2}} = \frac{\pi^2}{8}$

Arege Polar: $\frac{\pi^2/8}{\pi/2} = \frac{\pi}{4}$

4. $\vec{\gamma}(\theta) = \begin{pmatrix} R\cos\theta \\ R\sin\theta \end{pmatrix}$ $\quad 0 \le \theta \le \pi$

$\|\vec{\gamma}'(\theta)\| = R$

$\int_0^{\pi} (R\cos\theta) \cdot R \cdot d\theta = 0$

So Average x is 0

$\int_0^{\pi} (R\sin\theta) \cdot R\, d\theta = 2R^2$

So Average y is $\frac{2R^2}{\pi R} = \frac{2}{\pi} R$

Problem 2 : Line integral of Vector Fields
Compute the following line integral of vector fields:

1. $\vec{F} = \begin{pmatrix} x+y \\ 2y \end{pmatrix}$, $C : \vec{\gamma}(t) = (t, t^2), 0 \le t \le 1$ from $t=0$ to $t=1$
2. $\vec{F} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $C : \vec{\gamma}(t) = \begin{pmatrix} 3\cos t \\ 3\sin t \\ t \end{pmatrix}, 0 \le t \le 2\pi$ from $t=0$ to $t=2\pi$

1. $\int_0^1 \begin{pmatrix} t+t^2 \\ 2t^2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2t \end{pmatrix} dt = \int_0^1 (t + t^2 + 4t^3)\, dt = \frac{11}{6}$

2. $\int_0^{2\pi} \begin{pmatrix} 3\cos t \\ 3\sin t \\ t \end{pmatrix} \cdot \begin{pmatrix} -3\sin t \\ 3\cos t \\ 1 \end{pmatrix} dt = \int_0^{2\pi} t \cdot dt = 2\pi^2$

Problem 3 : Conservative Vector Field

1. Given $\vec{F} = \begin{pmatrix} 2xe^{xy} + x^2ye^{xy} \\ x^3e^{xy} + 2y \end{pmatrix}$, is $\vec{F}$ conservative or not? If so, find the potential function.

2. Given $\vec{F} = \begin{pmatrix} y \\ z \\ x \end{pmatrix}$, is $\vec{F}$ conservative or not?

3. Consider the vector field in 3.1, C is the upper half unit circle starting from $(-1, 0)$ to $(1, 0)$, compute the line integral of vector field.

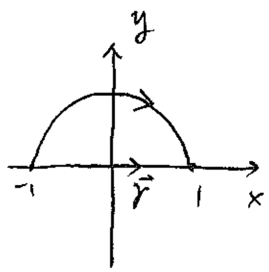
1. $P_y = 2x \cdot e^{xy} \cdot x + x^2 \cdot e^{xy} + x^2 \cdot y \cdot e^{xy} \cdot x$

$Q_x = 3x^2 \cdot e^{xy} + x^3 \cdot e^{xy} \cdot y$

$P_y = Q_x$, P and Q are both defined everywhere. So $\vec{F}$ is conservative

2. $P_y = 1 \neq Q_x$. no.

3.



Using $\vec{r}(t) = \begin{pmatrix} t \\ 0 \end{pmatrix}$ $\quad -1 \le t \le 1$

$$\int_{-1}^{1} \begin{pmatrix} 2t \\ t^3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} dt = \int_{-1}^{1} 2t \cdot dt = t^2 \Big|_{-1}^{1} = 0$$