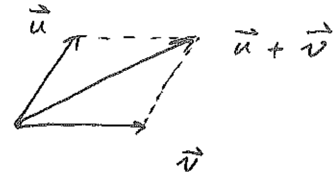
Vector Algebra

$+$:
$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{pmatrix}$$



$$(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$$

$\| \cdot \|$: $\quad \|\vec{u}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

scalor multiplication:
$$\lambda \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \lambda a_1 \\ \lambda a_2 \\ \lambda a_3 \end{pmatrix}$$

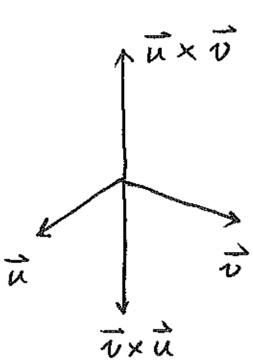
$$\lambda \cdot (\vec{v}_1 + \vec{v}_2) = \lambda \cdot \vec{v}_1 + \lambda \vec{v}_2$$
$$(\lambda_1 + \lambda_2) \cdot \vec{v} = \lambda_1 \vec{v} + \lambda_2 \vec{v}$$

dot product:
$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \cos\theta \qquad (\vec{u} \cdot \vec{v} = 0 \Leftrightarrow \vec{u} \perp \vec{v}) \ ☆$$

$$\vec{u} = a_1 \vec{e}_1 + a_2 \vec{e}_2 + a_3 \vec{e}_3 \qquad \vec{u} \cdot \vec{e}_i = a_i$$

cross product:
$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$



- right hand rule for direction
- $\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \sin\theta \quad (\vec{u} \times \vec{v} = \vec{0} \Leftrightarrow \vec{u} \parallel \vec{v})$