Homework 3, Math 401

due on Feb 15, 2021

Before you start, please read the syllabus carefully.

1. Let p be a prime number. Prove that group of order p are all isomorphic to C_p.
2. Prove that $S_n = \langle (1i) \in S_n \mid 2 \leq i \leq n \rangle$.
3. Find all group homomorphisms $f : (\mathbb{Z}, +) \to C_n$, and describe the associated $\ker(f)$. Prove that $\mathbb{Z}/\langle n \rangle \simeq C_n$ where $\langle n \rangle$ is the subgroup generated by n in $\mathbb{Z}$.
4. Recall $[G, G] := \langle a^{-1}b^{-1}ab \mid a \in G, b \in G \rangle$. Prove that $G/[G, G]$ is abelian.
5. Let $H \subset G$ be a subgroup of G. We say that H_1 is conjugate to H_2 if and only if $H_1 = gH_2g^{-1}$ for some $g \in G$. We denote it to be $H_1 \sim H_2$. Prove that conjugation is an equivalence relation among all subgroups of G.
6. Let G be a group. We say that g_1 is conjugate to g_2 if and only if $g_1 = \sigma g_2 \sigma^{-1}$ for some $\sigma \in G$. we denote it to be $g_1 \sim g_2$. Prove that conjugation is an equivalence relation among all elements of G.
7. Prove that the group $C_6 \simeq C_3 \times C_2$.
8. Prove that if $H \triangleleft S_n$ and $(12) \in H$, then $H = S_n$.