Homework 1, Math 3000

due on Jan 18, 2022

Before you start, please read the syllabus carefully.

1. Solve the following linear systems.

   (a)
   $$\begin{cases} 3x + 2y = 7 \\ 7x + 5y = 17 \end{cases}$$

   (b)
   $$\begin{cases} x + 2y + 3z = 6 \\ 2x + 3y + z = 6 \\ 3x + y + 2z = 6 \end{cases}$$

   (c)
   $$\begin{cases} x + 2y = 5 \\ 2x + y = 4 \\ x + y = 3 \end{cases}$$

   (d)
   $$\begin{cases} y + z = 0 \\ x + z = 1 \\ x + y + 2z = 1 \end{cases}$$

   (e)
   $$\begin{cases} y + z = 0 \\ x + z = 1 \\ x + y + 2z = 2 \end{cases}$$

2. Determine for what value of a b c and d the following linear systems have at least one solution. Then determine for what value of a b c and d the following linear systems have infinitely many solutions.

   (a)
   $$ax = 0.$$

   (b)
   $$ax = 1.$$

(c)

$$\begin{cases} ax + by = 0 \\ cx + dy = 0 \end{cases}$$

(d)

$$\begin{cases} ax + by = 1 \\ cx + dy = 2 \end{cases}$$

3. Fix the value of a, b, c and d. Prove that if (x_0, y_0) is a solution for the linear system in Ex. $2(c)$ and (x_1, y_1) is a solution for the linear system in Ex. $2(d)$, then $(x_0 + x_1, y_0 + y_1)$ is also a solution for $2(d)$.