

Homework 12, Math 3000

due on May 3, 2022

Before you start, please read the syllabus carefully. (You can use the calculator for this assignment.)

1. A mixture of 100 g H_2O is in the form of water or water vapor. Under a given status, the probability of water transforming to water vapor is 0.8, and the probability of water vapor transforming to water is 0.5. Compute the mass of water and water vapor respectively after a long time.
2. A rectangle has 4 vertexes A, B, C, D . A particle at each vertex has equal probability of walking to neighboring points (including itself). Compute the steady state for this Markov chain.
3. Solve the following differential equations:

(a)

$$\begin{cases} \frac{dx}{dt} = x - 2y; \\ \frac{dy}{dt} = 3x - 4y. \end{cases}$$

(b)

$$\begin{cases} \frac{dx}{dt} = 2x - 3y + z; \\ \frac{dy}{dt} = -3x - 2y - z \\ \frac{dz}{dt} = x - y + z. \end{cases}$$

4. Complete the squares for the following quadratic polynomials:

(a) $f(x, y) = x^2 + xy + y^2$;

(b) $f(x, y) = xy$;

(c) $f(x, y, z) = x^2 + y^2 - 2yz + xz + 3xy$.