Homework 3, Math 3000

due on Feb 1, 2022

Before you start, please read the syllabus carefully.

1. For the following set, determine whether V forms a vector space:
   (a) V is the set of continuous functions f on $\mathbb{R}$ where $f(x+2\pi) = f(x)$, where $+$ and scalar multiplication is the usual addition of two functions and usual multiplication by a number.
   (b) V is the set of real coefficients polynomials $f(x)$ with degree smaller or equal n and $f(0) = 0$, where $+$ and scalar multiplication are the usual addition of two polynomials and usual multiplication by a number.
   (c) V is the set of matrices of A in $M_{m\times n}(\mathbb{R})$ with $\sum_j A_{1j} = 1$, where $+$ and scalar multiplication are the usual addition of two matrices and usual multiplication of a matrix by a number.
   (d) V is the set of matrices of A in $M_{n\times n}(\mathbb{R})$ with $A_{ii} = 0$ for all i.
   (e) V is the set of vectors in the form of $\mathbf{u} = (x, y, 1-x-y)$ in $\mathbb{R}^3$, where $+$ and scalar multiplication is the usual addition and scalar multiplication of vectors.
2. For the set in Ex. 1, determine whether corresponding $W \subset V$ forms a subspace (i.e. W in 2 (a) is a subset in V in 1 (a)):
   (a) W is the subset of continuous functions f with $f(x) \geq 0$.
   (b) W is the subset of polynomials with $f'(0) = 0$.
   (c) W is the set of matrices in V with $A_{11} = 0$.
   (d) W is the set of matrices in V with $A_{11} = 0$.
   (e) W is the set of vectors in the form of $(t, t^2, 1-t-t^2)$.
3. For $\mathbf{u} = (2, 1, 0)$ and $\mathbf{v} = (1, 2, 1)$, prove that:
   (a) All vectors in $\mathbb{R}^3$ that are perpendicular to both $\mathbf{u}$ and $\mathbf{v}$ form a subspace.
   (b) All vectors that are linear combination of $\mathbf{u}$ and $\mathbf{v}$ form a subspace.
4. For $\mathbf{u} = (2, 1, 0)$, $\mathbf{v} = (1, 2, 1)$, $\mathbf{w} = (1, 2, 3)$, determine whether $\mathbf{u}, \mathbf{v}, \mathbf{w}$ span R^3.
5. For $\mathbf{u} = (1, 1, 0)$, $\mathbf{v} = (0, 1, 1)$, $\mathbf{w} = (1, 0, 1)$, determine whether $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are linearly independent.
6. Given a vector space V and two subspaces W_1 and W_2, define $W_1 + W_2$ to be $\{u + v \mid u \in W_1, v \in W_2\}$. Prove that $W_1 + W_2$ is also a subspace in V.