

# Homework 4, Math 3000

due on Feb 8, 2022

Before you start, please read the syllabus carefully.

1. Determine whether the following list of vectors are: 1) linearly independent 2) spanning the vector space.

(a)  $\{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$  in  $\mathbb{R}^3$

(b)  $\{1, x - 1, (x - 1)^2, (x - 1)^3\}$  in the vector space of polynomials with degree smaller or equal to 3

(c)  $\{(1, 1, -2), (1, -2, 1), (-2, 1, 1)\}$  in the vector space  $W := \{(x, y, z) \mid x + y + z = 0\}$

(d)  $\left\{ \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \right\}$  in the vector space  $M_{2 \times 2}(\mathbb{R})$ .

2. Give a basis of the following vector spaces and determine its dimension:

(a)  $V := \{A\mathbf{x} \mid \mathbf{x} \in \mathbb{R}^3\}$  where

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

(b)  $V := \{\mathbf{x} \in \mathbb{R}^4 \mid A\mathbf{x} = 0\}$  where

$$A = \begin{pmatrix} 1 & 1 & 2 & 0 \\ 2 & 0 & 1 & 1 \end{pmatrix}$$

(c)  $V := \{\text{polynomials } P(x) \mid P'(1) = P''(1) = 0, \deg(P(x)) \leq 4\}$ .

(d)  $V$  is the intersection of planes  $2x + y + z = 0$  and  $x + 2y + z = 0$  in  $\mathbb{R}^3$ .

(e)  $V$  is the set of symmetric matrices in  $M_{2 \times 2}(\mathbb{R})$ . ( $A$  is symmetric means  $A_{ij} = A_{ji}$  for all  $i, j$ )