Homework 5, Math 3000

due on Feb 15, 2022

Before you start, please read the syllabus carefully.

1. For the following maps: 1) prove it is linear map; 2) write down the matrix for the following linear maps (under the prescribed basis of V).
   (a) $f : \mathbb{R}^3 \to \mathbb{R}^4$ where $f((1,0,0)) = (1,2,2,1)$, and $f((0,1,0)) = (0,1,0,1)$ and $f((0,0,1)) = (1,0,1,0)$. (using the standard basis of $\mathbb{R}^n$)
   (b) $f : V \to V$ where V is the set of polynomials with degree smaller or equal to 3 and $f(P(t)) = P'(t)$ is the derivative map. (using the basis $1, t, t^2, t^3$)
   (c) $f : V \to V$ where V is the set of polynomials with degree smaller or equal to 3 and $f(P(t)) = 2 \cdot P(t-1)$. (using the basis $1, t, t^2, t^3$)
   (d) $f : V \to \mathbb{R}$ where V is the set of polynomials with degree smaller or equal to 3 and $f(P(t)) = P'(1)$. (using the basis $1, t, t^2, t^3$)
   (e) $f := V \to V$ where V is space of 2×2 matrices and $f(A) = A + A^T$. (using the standard basis E_{ij} for matrices)
2. For each of the linear transformation in Ex. 1, compute the basis for the subspaces $\mathrm{Ker}(f)$ and $\mathrm{Im}(f)$ and determine their dimension.