Homework 6, Math 3000

due on March 1, 2022

Before you start, please read the syllabus carefully.

1. Let
$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}.$$

    Compute the following matrix operations

    (a) AB

    (b) A^{-1}

    (c) B^{-1} (Does this exist?)

    (d) $B^{-1}A^{-1}$ (Does this exist?)

    (e) A^2, B^2

    (f) A^3, B^2

    (g) $(A+B)^{-1}$

2. The standard basis for $\mathbb{R}^3$ is $E = \{(1,0,0),(0,1,0),(0,0,1)\}$. The new basis is $E' = \{(1,0,0),(1,1,0),(1,1,1)\}$.

    (a) Write down the base change matrix from E to E'.

    (b) Let T_1 be a linear map represented by A (see Ex. 1) under basis E, find the matrix representing T_1 under basis E'.

    (c) Let T_2 be a linear map represented by B (see Ex. 1) under basis E, find the matrix representing T_2 under basis E'.

    (d) Let $T_3 := T_1 \circ T_2 : \mathbb{R}^3 \to \mathbb{R}^3$ be the composition of T_1 and T_2, find the matrix representing T_3 under basis E and E'.

3. Let V be the vector space of polynomials with degree ≤ 3. The standard basis for V is $E = \{1, t, t^2, t^3\}$. The new basis is $E' = \{1, (t-1), (t-1)^2, (t-1)^3\}$.

    (a) Write down the matrix for the linear map of derivative using basis E.

    (b) Write down the base change matrix from E to E'.

    (c) Write down the matrix for the linear map of derivative using basis E'.